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Enhancing Possibilistic Fuzzy Goal Programming Approach for Solving Multi Objective Minimum Cost Flow Problems Coefficients

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Abstract

This study investigates a multi-objective minimum cost flow with probabilistic objective function coefficients (Poss- MOMCF). Under using the $\alpha-$cut set of the possibilistic variables, the Poss-MOMCF problem is converted into the corresponding (α-MOMCF) and hence into the (P-MOMCF) problem. A necessary and sufficient condition for investigating the $\alpha-$possibly optimal solution is established. A fuzzy goal programming approach is applied to obtain the $\alpha-$parametric optimal compromise solution. The stability set of the first kid under the concept of $\alpha-$possibly optimal solution is characterized and analyzed without differentiability. Finally, a numerical example is given in the sake of the paper to clarify the methodology.

Keywords: Optimization problem, Minimum cost flow, Multi-objective optimization, Possibilistic variables, Fuzzy goal programming approach, $\alpha-$possibly optimal solution, Goal programming, Decision maker, Compromise solution, Parametric analysis.

1 | Introduction

The Minimum Cost Flow (MCF) problem is a general form of the network flow problem, which aims to determine the east coast of the shipment of a commodity during a capacitated network to achieve demands at certain nodes from supplies at other nodes. The study of MCF can be investigated to enormous other network problems, for example, maximum flow, assignment, shortest path, transportation, transshipment problems, multi-stage production inventory planning, nurse scheduling, project assignment, mold allocation,

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college course assignment, and automobile routing [1]-[4]. Hu et al. [5] proposed an algorithm that holds the complementarity slackness at each iteration with the help of a dual approach that is node by updating node potential iteratively; they found an augmenting path.

Multi-objective MCF problems have been studied in the literature [6], [7]. Bazaraa et al. [8] applied the parametric analysis in large-scale linear programming. Steuer [9] decomposed the parametric space of convex combination parametric programming using the parametric analysis. Luhandjula [10] studied the multi-objective linear problem having coefficients represented by possibilistic data. Lee and Moore [11] and Hemaida and Kwak [12] reviewed the Multi-Objective Transportation Problem (MOTP) and applied Goal Programming (GP) to obtain a satisfactory solution. Tamiz et al. [13] and Romero [14] discussed the shortcomings of GP. Many authors applied a fuzzy programming approach to solving MOTP [15]-[22]. Cui et al. [23] proposed a novel general MCF model for optimizing the distribution pattern of evacuation flow and rescue flow on the same network by introducing the conflict cost.

This paper introduces a multi-objective MCF problem with probabilistic variables. The parametric study corresponding to the $\alpha -$possibly optimal solution is defined and determined without differentiability.

The above literature analysis clearly shows that the proposed study is novel, more generalized, and flexible compared to existing relevant literature. It has the distinction due to the inclusion of the following features for the first time in literature:

I. Possibilistic multi-objective MCF.

II. Possibilistic in all of the coefficients of the objective functions.

III. α-pareto optimal solution-based scenario.

IV. The stability set of the first kind without differentiability.

The rest of the paper is outlined as follows.

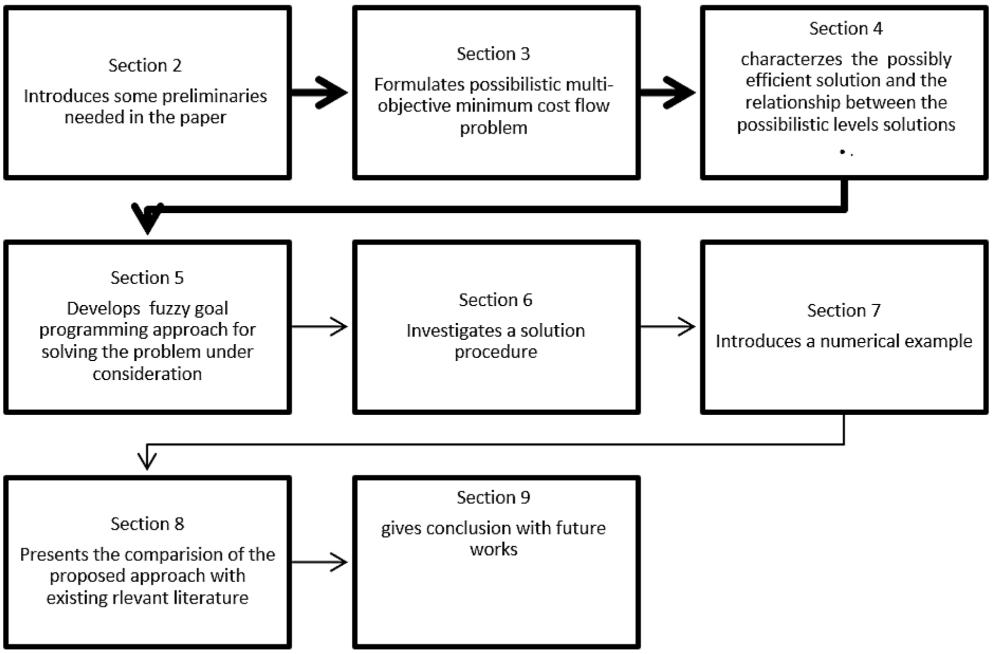

Fig. 1. Layout of remaining paper.

2 | Preliminaries

This section introduces some basic concepts and results related to possibilistic variables and its α −level set, possibilistic distribution, and support.

Definition 1 ([24], [25]). A possibilistic variable u on V is a variable characterized by a possibility distribution$\mu_u(v)$, which means that if u is a variable taking values in V, then μ_u corresponding to u may be viewed as a fuzzy constraint. Such a distribution is characterized by a possibility distribution function $\mu_u: V \longrightarrow [0,1]$, which is associated with each $v \in V$, the degree of compatibility of u with the realization $v \in V$. If V is a Cartesian product of $V_1, V_2, \dots, V_n$, then $\mu_u(v_1, v_2, \dots, v_n)$ is an n −ary possibility distribution, i.e.,

$$\mu_u(v) = \left(\mu_{u_1}(v_1), \mu_{u_2}(v_2), \dots, \mu_{u_n}(v_n)\right).$$

Definition 2. The α −level set of possibilistic variable u is

$$u_\alpha = \{v \in V: \mu_u(v) \geq \alpha\}.$$

Definition 3 ([24]). A possibility distribution μ_u on V is said to be convex if

$$\mu_u(\gamma v^1 + (1-\gamma)v^2) \geq \min\left(\mu_u(v^1), \mu_u(v^2)\right), \text{for all } v^1,\ v^2 \in V, \gamma \in [0,1].$$

Definition 4 ([24]). The support of a possibilistic variable u is

$$\text{Supp } (u) = \left\{v \in V: \sup_{v \in N_\delta(y)} \mu_u(v) > 0, \text{for all } \delta > 0\right\}, \text{ where } N_\delta(y) = \{v \in V \in: \|v-y\| < \delta\}.$$

3 | Problem Statement and Solution Concepts

Consider the following possibilistic multi-objective MCF problem.

(Poss MOMCF) $\min \tilde{F}_r(x, \tilde{c}^r) = \sum_{(i,j)\in M} \tilde{c}_{ij}^r x_{ij}, r = 1, 2, \dots, K,$

s.t.

$$x \in X = \left\{\begin{array}{lll} \sum\limits_{j:\,(i,j)\in M} x_{ij} - \sum\limits_{k:(k,i)\in M} x_{li} = b(i), & & \text{for all } i \in V, \\ x_{ij} \in U_{ij}, & \text{for all } (i,j) \in M, x_{ij} \geq 0, & \text{for all } (i,j) \in M, \end{array}\right\}$$

where

M: The set of arcs (i,j),

V: The set of nodes,

x_{ij}: The decision variable representing the flow through the arc (i,j),

$U_{ij} = [l_{ij}, u_{ij}]$: Capacity of arc (i,j),

$\tilde{c}_{ij}^r$: The possibilistic penalty per unit of flow through the arc (i,j) in the $\tilde{c}_{ij}^r$ objective function $r = 1, 2, \dots, K$,

$b(i)$: The net flow generated at node i, the values of $b(i)$ being positive, zero, or negative classifies node i as a supply node, transshipment node, or demand node, respectively.

It is noted that the parameters $\tilde{c}_{ij}^{K}$ is vectors of possibilistic variables on $\mathbb{R}$, which is characterized by possibility distributions $\mu_{\tilde{c}_{ij}^{r}}$. It is assumed that all possibility distributions in the Poss MOMCF problem are convex cones having compact supports and $u_0 = \mathrm{supp}(u)$.

Definition 5. $x^* \in G$ is an α −possibly efficient solution for Poss MOMCF if there is no $x \in G$ such that

$$\mathrm{Poss}\begin{pmatrix} F_1(x,\tilde{c}^1) \le F_1(\hat{x},\tilde{c}^1), F_2(x,\tilde{c}^2) \le F_2(\hat{x},\tilde{c}^2), \\ \dots, F_{r-1}(x,\tilde{c}^{r-1}) \le F_{r-1}(\hat{x},\tilde{c}^{r-1}), F_r(x,\tilde{c}^r) \le F_r(\hat{x},\tilde{c}^r), \\ F_{r+1}(x,\tilde{c}^{r+1}) \le F_{r+1}(\hat{x},\tilde{c}^{r+1}), \dots, F_K(x,\tilde{c}^K) \le F_K(\hat{x},\tilde{c}^K) \end{pmatrix} \ge \alpha. \quad (1)$$

Based on the extension principle, we have

$$\mathrm{Poss}\begin{pmatrix} F_1(x,\tilde{c}^1) \le F_1(\hat{x},\tilde{c}^1), F_2(x,\tilde{c}^2) \le F_2(\hat{x},\tilde{c}^2), \\ \dots, F_{r-1}(x,\tilde{c}^{r-1}) \le F_{r-1}(\hat{x},\tilde{c}^{r-1}), F_r(x,\tilde{c}^r) \le F_r(\hat{x},\tilde{c}^r), \\ F_{r+1}(x,\tilde{c}^{r+1}) \le F_{r+1}(\hat{x},\tilde{c}^{r+1}), \dots, F_K(x,\tilde{c}^K) \le F_K(\hat{x},\tilde{c}^K) \end{pmatrix}$$
$$= \sup_{(c^1, c^2, \dots, c^K)\in E} \min\begin{pmatrix} \mu_{\tilde{c}^1}(c^1), \mu_{\tilde{c}^2}(c^2), \dots, \mu_{\tilde{c}^{r-1}}(c^{r-1}), \\ \mu_{\tilde{c}^r}(c^r), \ \mu_{\tilde{c}^{r+1}}(c^{r+1}), \dots, \ \mu_{\tilde{c}^K}(c^K) \end{pmatrix}, \quad (2)$$

where

$$E = \begin{Bmatrix} (c^1, c^2, \dots, c^K): F_1(x,\tilde{c}^1) \le F_1(\hat{x},\tilde{c}^1), F_2(x,\tilde{c}^2) \le F_2(\hat{x},\tilde{c}^2), \\ \dots, F_{r-1}(x,\tilde{c}^{r-1}) \le F_{r-1}(\hat{x},\tilde{c}^{r-1}), F_r(x,\tilde{c}^r) \le F_r(\hat{x},\tilde{c}^r), \\ F_{r+1}(x,\tilde{c}^{r+1}) \le F_{r+1}(\hat{x},\tilde{c}^{r+1}), \dots, F_K(x,\tilde{c}^K) \le F_K(\hat{x},\tilde{c}^K) \end{Bmatrix}. \quad (3)$$

$\mu_{\tilde{c}^r}, r = 1, 2, \dots, K$ are arcs $K(i,j)$ possibly distributions.

4 | Characterization of α −Possibly Efficient Solution for Poss MOMCF Problem

For investigating the α −possibly efficient solutions for the Poss MOMCF problem, let us consider the α −parametric multi-objective MCF problem

$$(\alpha\text{−PMOMCF}) \quad \min F_r(x,c^r) = \sum_{(i,j)\in M} c_{ij}^r x_{ij}, \ r = 1, 2, \dots, K,$$

s.t.

$$x \in X = \begin{Bmatrix} \sum_{j:\,(i,j)\in M} x_{ij} - \sum_{k:(k,i)\in M} x_{li} = b(i), & \text{for all } i \in V, \\ x_{ij} \in U_{ij}, \quad \text{for all } (i,j) \in M, x_{ij} \ge 0, & \text{for all } (i,j) \in M, c_{ij}^r \in (\tilde{c}_{ij}^r)_\alpha, \end{Bmatrix} \quad (4)$$

where, $(\tilde{c}_{ij}^r)_\alpha$ is the α −cut of the possibilistic variable c_{ij}^r. Based on the convexity assumption $\mu_{\tilde{c}_{ij}^r}(c_{ij}^r)$, (i,j) is arc, $r = \overline{1,K}$ are real intervals denoted by $\left[(c_{ij}^{r-})_\alpha, (c_{ij}^{r+})_\alpha\right]$. Let φ_α^r be the set of arcs (i,j) with $c_{ij}^r \in \left[(c_{ij}^{r-})_\alpha, (c_{ij}^{r+})_\alpha\right]$, $r = \overline{1,K}$. The α −PMOMCF problem can be rewritten as

$$\min F_r(x,c^r) = \sum_{(i,j)\in M} c_{ij}^r x_{ij}, \ r = 1, 2, \dots, K,$$
s.t. $\quad (5)$
$$x \in X, \text{ and } c^r \in \varphi_\alpha^r, r = \overline{1,K}.$$

Problem (5) can be rewritten as

$$\min F_r(x,c^r) = \sum_{(i,j)\in M}(c_{ij}^{r-}(\tau) + \tau\, c_{ij}^{r-}(\tau))x_{ij}, r = 1,2,\dots,K,$$
$$\text{s.t.} \tag{6}$$
$$x \in X,\ \text{and}\ c^r \in \varphi_\alpha^r, r = \overline{1,K}, \qquad \tau \in [0,1].$$

Definition 6. $x^* \in G$ is an α −parametric efficient solution for α −PMOMCF problem if there is no $x \in G$ and $c^r \in \varphi_\alpha^r$ such that $F_r(x,c^r) \le F_r(x^*,c^r)$; $\forall r = \overline{1,K}$ and $F_r(x,c^r) < F_r(x^*,c^r)$ holds for at least one r.

Definition 7 ([26]). A feasible vector $Y^\circ \in X$ is said to be α −parametric compromise solution of α −PMOMCF if and only if $Y^\circ \in H$ and $F(Y) \le \wedge_{Y\in X} F(Y)$, where $\wedge$ stands for the minimum, and H is the set of α −parametric efficient solutions.

Definition 8 ([15]). If the α −parametric compromise solution satisfies the decision makers' preference, the solution is called α −preferred parametric compromise solution.

Theorem 1. $x^* \in G$ is an α −possibly efficient solution for the Poss MOMCF problem if and only if α −parametric efficient solution for α −PMOMCF problem.

Proof: (see [24]).

5 | Fuzzy Goal Programming Approach for Solving Problem (5)

Based on the three concepts of fuzzy goals (G), fuzzy constraints (C), and fuzzy decision (D) introduced by Bellman and Zadeh [27], the fuzzy decision is defined as

$$D = C \cap G. \tag{7}$$

Then

$$\mu_D(x) = \min(\mu_C(x), \mu_G(x)). \tag{8}$$

With the membership *Function (8)*, let us describe the fuzzy goals for the problem under study. The linear membership function (MP) [28] is given by

$$\mu_r(F_r(x,c^r)) = \begin{cases} 0, & F_r(x,c^r) \le L_r, \\ \frac{U_r - F_r(x,c^r)}{U_r - L_r}, & L_r < F_r(x,c^r) < U_r, \\ 1, & F_r(x,c^r) \ge U_r, \end{cases} \tag{9}$$

where, L_r, and U_r are the lower and upper bounds of $F_r(x,c^r)$, $L_r \ne U_r$, and can be calculated as

$$L_r = \min_x F_r(x,c^r),\ \text{and}\ U_r = \max_x F_r(x,c^r), \quad r = 1,2,\dots,K. \tag{10}$$

By applying the fuzzy *Decision (8)* and membership *Function (9)*, α −PMOMCF can be rewritten as

$$\max \min_{r=\overline{1,K}} \left(\mu_r(F_r(x,c^r))\right),$$
$$\text{s.t.} \tag{11}$$
$$x \in X,\ \text{and}\ c^r \in \varphi_\alpha^r, r = \overline{1,K}.$$

Problem (11) can be converted into well-defined linear programming using the auxiliary variable ϑ as

$$\max \vartheta \tag{12}$$

$$
\begin{aligned}
&\text{s.t.}\\
&\vartheta \le \mu_r\big(F_r(x, c^r)\big), r = \overline{1, K},\\
&x \in X,\ \text{and } c^r \in \varphi_\alpha^r, r = \overline{1, K}.
\end{aligned}
$$

In order to formulate *Problem (12)* as a GP [29], we introduce the negative and positive deviational variables.

$$F_r(x, c^r) - v_r^+ + v_r^- = G_r, r = \overline{1, K}, \tag{13}$$

where, G_r is the aspiration level of the objective function r. Now, *Problem (12)* is reformulated as a mixed integer GP problem as

$$
\begin{aligned}
&\max \vartheta\\
&\text{s.t.}\\
&\vartheta \le \mu_r\big(F_r(x, c^r)\big), r = \overline{1, K},\\
&x \in X,\ \text{and } c^r \in \varphi_\alpha^r, r = \overline{1, K},\\
&F_r(x, c^r) - v_r^+ + v_r^- = G_r,\\
&v_r^-, v_r^+ \ge 0, r = \overline{1, K}, 0 \le \vartheta \le 1.
\end{aligned} \tag{14}
$$

6 | Solution Procedure

In this section, a solution procedure for solving the Poss MOMCF problem can be summarized in the following steps:

Step 1. Consider the Poss MOMCF problem.

Step 2. Solve each one of the objective function and continue this process K times. If all the resulting solutions are equal, select one of them and go to Step 5.

Step 3. Define the membership function of every one of the objectives and also the aspiration level.

Step 4. Construct *Problem (13)*, then solve it using any computer package (Say, GAMS).

Step 5. Stop and then determine the stability set of the first kind $S(x^\circ, c^\circ)$ as

$$
\begin{aligned}
&\omega_{ij}\left(c_{ij}^r - \left(c_{ij}^{r+}\right)_\alpha\right) = 0, \text{for all arc } (i, j), r = \overline{1, K},\\
&\varphi_{ij}\left(\left(c_{ij}^{r-}\right)_\alpha - c_{ij}^r\right) = 0, \text{for all arc } (i, j), r = \overline{1, K}.
\end{aligned}
$$

7 | Numerical Example

Consider the following problem:

$$
\begin{aligned}
&\min F_1(x, c^1) = \textstyle\sum_{(i,j)\in M} \tilde{c}_{ij}^1 x_{ij},\\
&\min F_2(x, c^2) = \sum_{(i,j)\in M} \tilde{c}_{ij}^2 x_{ij},\\
&\text{s.t.}\\
&x_{12} + x_{13} = 10,\\
&x_{24} + x_{25} - x_{12} = 0,
\end{aligned} \tag{15}
$$

$x_{34} + x_{35} - x_{13} = 20,$

$x_{45} - x_{24} - x_{34} = -15,$

$-x_{25} - x_{35} - x_{45} = -15,$

$x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10],$

$x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50].$

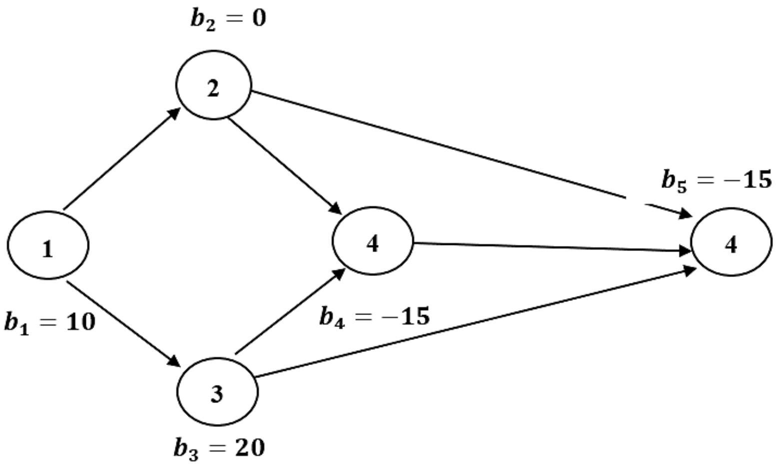

Fig. 2. A network with 5 nodes and 7 arcs ([6]).

The possibilistic variables $\tilde{c}_{ij}^1$, and $\tilde{c}_{ij}^2$ are represented by a possibility distributions $\mu_{\tilde{c}_{ij}^1}(.)$, and $\mu_{\tilde{c}_{ij}^2}(.)$ In *Fig. 1* and *Fig. 2*. The supports of the possibilistic variables $\tilde{c}_{ij}^1$, and $\tilde{c}_{ij}^2$ are[3,12], and $[2, 10]$. Hence, for the parametric functions$0 \leq \tau \leq 1$, the supports are

Supp $(\tilde{a}_1) = 1 + 4\vartheta$, $\quad \mu_{\tilde{a}_1}(1) = \mu_{\tilde{a}_1}(5) = 0,$

Supp $(\tilde{a}_2) = 10 - 4\vartheta$, $\quad \mu_{\tilde{a}_2}(10) = \mu_{\tilde{a}_2}(6) = 0.$

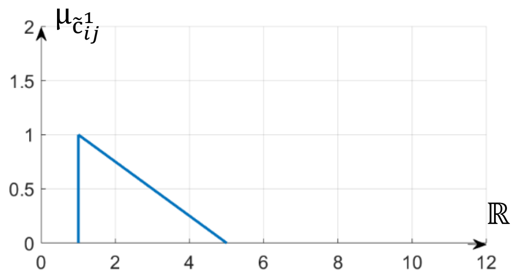

Fig. 3. Possibility distributions $\mu(.)$ for $\tilde{c}_{ij}^1$.

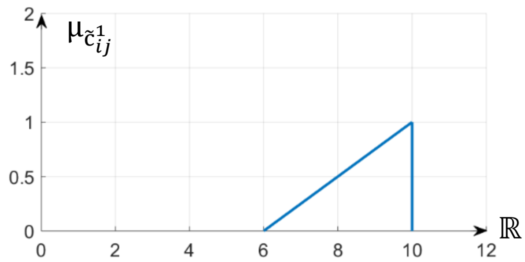

Fig. 4. Possibility distributions $\mu(.)$ for $\tilde{c}_{ij}^2$.

$\mathrm{Supp}\ (\tilde{c}_{12}^{1}) = 3 + 4\tau, \quad \mu_{\tilde{c}_{12}^{1}}(3) = \mu_{\tilde{c}_{12}^{1}}(7) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{13}^{1}) = 6 - 2\tau, \quad \mu_{\tilde{c}_{13}^{1}}(4) = \mu_{\tilde{c}_{13}^{1}}(6) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{24}^{1}) = 5 + 4\tau, \quad \mu_{\tilde{c}_{24}^{1}}(5) = \mu_{\tilde{c}_{24}^{1}}(9) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{25}^{1}) = 7 + \tau, \quad \mu_{\tilde{c}_{13}^{1}}(7) = \mu_{\tilde{c}_{13}^{1}}(8) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{34}^{1}) = 9 + 2\tau, \quad \mu_{\tilde{c}_{34}^{1}}(9) = \mu_{\tilde{c}_{34}^{1}}(11) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{35}^{1}) = 10 + 2\tau, \quad \mu_{\tilde{c}_{35}^{1}}(10) = \mu_{\tilde{c}_{35}^{1}}(12) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{45}^{1}) = 1 + \tau, \quad \mu_{\tilde{c}_{45}^{1}}(1) = \mu_{\tilde{c}_{45}^{1}}(2) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{12}^{2}) = 13 - 4\tau, \quad \mu_{\tilde{c}_{12}^{2}}(13) = \mu_{\tilde{c}_{12}^{2}}(9) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{13}^{2}) = 4 + 4\tau, \quad \mu_{\tilde{c}_{13}^{2}}(4) = \mu_{\tilde{c}_{13}^{2}}(8) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{24}^{2}) = 7 - 4\tau, \quad \mu_{\tilde{c}_{24}^{2}}(7) = \mu_{\tilde{c}_{24}^{2}}(3) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{25}^{2}) = 4 + 4\tau, \quad \mu_{\tilde{c}_{25}^{2}}(4) = \mu_{\tilde{c}_{25}^{2}}(8) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{34}^{2}) = 7 - 2\tau, \quad \mu_{\tilde{c}_{34}^{2}}(5) = \mu_{\tilde{c}_{34}^{2}}(7) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{35}^{2}) = 4 + 2\tau, \quad \mu_{\tilde{c}_{35}^{2}}(4) = \mu_{\tilde{c}_{35}^{2}}(6) = 0,$

$\mathrm{Supp}\ (\tilde{c}_{45}^{2}) = 10 - 4\tau, \quad \mu_{\tilde{c}_{45}^{2}}(6) = \mu_{\tilde{c}_{45}^{2}}(10) = 0.$

$$
\begin{aligned}
&\mathrm{Min}\ F_1(\tau) = (3 + 4\tau)x_{12} + (6 - 2\tau)x_{13} + (5 + 4\tau)x_{24} + (7 + \tau)x_{25} + (9 + 2\tau)x_{34} \\
&\quad +(10 + 2\tau)x_{35} + (1 + \tau)x_{45}, \\
&\mathrm{Min}\ F_2(\tau) = (13 - 4\tau)x_{12} + (4 + 4\tau)x_{13} + (7 - 4\tau)x_{24} + (4 + 4\tau)x_{25} + (7 - 2\tau)x_{34} \\
&\quad +(4 + 2\tau)x_{35} + (10 - 4\tau)x_{45}, \\
&\mathrm{s.t.} \\
&x_{12} + x_{13} = 10, \\
&x_{24} + x_{25} - x_{12} = 0, \\
&x_{34} + x_{35} - x_{13} = 20, \\
&x_{45} - x_{24} - x_{34} = -15, \\
&-x_{25} - x_{35} - x_{45} = -15, \\
&x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10], \\
&x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50], \text{ and } \tau \in [0,1].
\end{aligned} \tag{16}
$$

$L_1 = 265,\ U_1 = 375, L_2 = 250,\ U_1 = 425,$

At $\tau = 0$, the GP for the problem becomes

$$
\begin{aligned}
&\max \vartheta \\
&\mathrm{s.t.} \\
&3x_{12} + 6x_{13} + 5x_{24} + 7x_{25} + 9x_{34} + 10x_{35} + x_{45} + \vartheta 110 \leq 375, \\
&13x_{12} + 4x_{13} + 7x_{24} + 4x_{25} + 7x_{34} + 4x_{35} + 10x_{45} + \vartheta 175 \leq 425, \\
&x_{12} + x_{13} = 10, \\
&x_{24} + x_{25} - x_{12} = 0,
\end{aligned} \tag{17}
$$

$x_{34} + x_{35} - x_{13} = 20,$

$x_{45} - x_{24} - x_{34} = -15,$

$-x_{25} - x_{35} - x_{45} = -15,$

$x_{12} \in [0, 20], x_{13} \in [0, 10], x_{24} \in [0, 20], x_{25} \in [0,10],$

$x_{34} \in [0, 30], x_{35} \in [0,25], x_{45} \in [0, 50],$

$3x_{12} + 6x_{13} + 5x_{24} + 7x_{25} + 9x_{34} + 10x_{35} + x_{45} - v_1^+ + v_1^- = 265,$

$13x_{12} + 4x_{13} + 7x_{24} + 4x_{25} + 7x_{34} + 4x_{35} + 10x_{45} - v_2^+ + v_2^- = 250,$

$v_1^+, v_1^-, v_2^+, v_2^- =$ and $\vartheta \in [0,1]$.

Using the GINO software, the optimal compromise solution is

$$\overset{\circ}{x}_{12} = \overset{\circ}{x}_{24} = 8.56, \overset{\circ}{x}_{13} = 1.44, \overset{\circ}{x}_{25} = \overset{\circ}{x}_{45} = 0, \overset{\circ}{x}_{34} = 6.44, \overset{\circ}{x}_{35} = 15, v_1^+ = 20.11,$$

$$v_1^- = v_2^- = 0, v_2^+ = 32, \vartheta^\circ = 0.82.$$

To determine the stability set $S\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right)$,

$$\omega_{12}^1(3 - (c_{12}^{1+})_0) = 0, \omega_{13}^1(6 - (c_{13}^{1+})_0) = 0, \omega_{24}^1(5 - (c_{24}^{1+})_0) = 0, \omega_{25}^1\left(7 - \left(c_{25}^{1+}\right)_0\right) = 0,$$

$$\omega_{34}^1(9 - (c_{34}^{1+})_0) = 0, \omega_{35}^1\left(10 - \left(c_{35}^{1+}\right)_0\right) = 0, \omega_{45}^1\left(1 - \left(c_{45}^{1+}\right)_0\right) = 0, \omega_{12}^2(13 - (c_{12}^{2+})_0) = 0,$$

$$\omega_{13}^2(4 - (c_{13}^{2+})_0) = 0, \omega_{24}^2(7 - (c_{24}^{2+})_0) = 0, \omega_{25}^2\left(4 - \left(c_{25}^{2+}\right)_0\right) = 0, \omega_{34}^2(7 - (c_{34}^{2+})_0) = 0, \omega_{35}^2\left(4 - \left(c_{35}^{2+}\right)_0\right) = 0, \omega_{45}^2\left(10 - \left(c_{45}^{2+}\right)_0\right) = 0,$$

$$\omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1, \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 \geq 0.$$

We get $I_1 \subseteq \{1, 2\}$.

For $I_1 = \emptyset, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1, \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 = 0$. Then

$$S_{I_1}\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right) = \left\{\begin{array}{c} c_{ij}^r: (c_{12}^{1+})_0 \geq 3, (c_{13}^{1+})_0 \geq 6, (c_{24}^{1+})_0 \geq 5, \\ \left(c_{25}^{1+}\right)_0 \geq 7, (c_{34}^{1+})_0 \geq 9, \left(c_{35}^{1+}\right)_0 \geq 10, \left(c_{45}^{1+}\right)_0 \geq 1, (c_{12}^{2+})_0 \geq 13, \\ (c_{13}^{2+})_0 \geq 4, (c_{24}^{2+})_0 \geq 7, \left(c_{25}^{2+}\right)_0 \geq 4, (c_{34}^{2+})_0 \geq 7, \left(c_{35}^{2+}\right)_0 \geq 4, \left(c_{45}^{2+}\right)_0 \geq 10 \end{array}\right\}.$$

For $I_2 = \{1\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 > 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 = 0$. Then

$$S_{I_2}\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right) = \left\{\begin{array}{c} c_{ij}^r: (c_{12}^{1+})_0 = 3, (c_{13}^{1+})_0 = 6, (c_{24}^{1+})_0 = 5, \\ \left(c_{25}^{1+}\right)_0 = 7, (c_{34}^{1+})_0 = 9, \left(c_{35}^{1+}\right)_0 = 10, \left(c_{45}^{1+}\right)_0 = 1, (c_{12}^{2+})_0 \geq 13, \\ (c_{13}^{2+})_0 \geq 4, (c_{24}^{2+})_0 \geq 7, \left(c_{25}^{2+}\right)_0 \geq 4, (c_{34}^{2+})_0 \geq 7, \left(c_{35}^{2+}\right)_0 \geq 4, \left(c_{45}^{2+}\right)_0 \geq 10 \end{array}\right\}.$$

For $I_3 = \{2\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 = 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 > 0$. Then

$$S_{I_3}\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right) = \left\{\begin{array}{c} c_{ij}^{r}: (c_{12}^{1+})_0 \geq 3, (c_{13}^{1+})_0 \geq 6, (c_{24}^{1+})_0 \geq 5, \\ \left(c_{25}^{1+}\right)_0 \geq 7, (c_{34}^{1+})_0 \geq 9, \left(c_{35}^{1+}\right)_0 \geq 10, \left(c_{45}^{1+}\right)_0 \geq 1, (c_{12}^{2+})_0 = 13, \\ (c_{13}^{2+})_0 = 4, (c_{24}^{2+})_0 = 7, \left(c_{25}^{2+}\right)_0 = 4, (c_{34}^{2+})_0 = 7, \left(c_{35}^{2+}\right)_0 = 4, \left(c_{45}^{2+}\right)_0 = 10 \end{array}\right\}.$$

For $I_4 = \{1,2\}, \omega_{12}^1, \omega_{13}^1, \omega_{24}^1, \omega_{25}^1, \omega_{34}^1, \omega_{35}^1, \omega_{45}^1 > 0; \omega_{12}^2, \omega_{13}^2, \omega_{24}^2, \omega_{25}^2, \omega_{34}^2, \omega_{35}^2, \omega_{45}^2 > 0$. Then

$$S_{I_4}\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right) = \left\{\begin{array}{c} c_{ij}^{r}: (c_{12}^{1+})_0 = 3, (c_{13}^{1+})_0 = 6, (c_{24}^{1+})_0 = 5, \\ \left(c_{25}^{1+}\right)_0 = 7, (c_{34}^{1+})_0 = 9, \left(c_{35}^{1+}\right)_0 = 10, \left(c_{45}^{1+}\right)_0 = 1, (c_{12}^{2+})_0 = 13, \\ (c_{13}^{2+})_0 = 4, (c_{24}^{2+})_0 = 7, \left(c_{25}^{2+}\right)_0 = 4, (c_{34}^{2+})_0 = 7, \left(c_{35}^{2+}\right)_0 = 4, \left(c_{45}^{2+}\right)_0 = 10 \end{array}\right\}.$$

Hence

$$S\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right) = \bigcup_{P=1}^{4} S_{I_p}\left(\overset{\circ}{x}_{12}, \overset{\circ}{x}_{24}, \overset{\circ}{x}_{13}, \overset{\circ}{x}_{25}, \overset{\circ}{x}_{45}, \overset{\circ}{x}_{34}, \overset{\circ}{x}_{35}, c_{ij}^{1^\circ}, c_{ij}^{2^\circ}\right).$$

8 | Discussion

In this section, the proposed study is compared with some existing relevant literature to carve out the advantageous aspect of the proposed research. *Table 1* presents this comparison under specific parameters. It's evident that the result obtained by the proposed approach is less than that obtained by Alharbi et al. [30].

Table 1. Comparisons of different researcher's contributions.

Author's Name	α −Efficient Solution	α −Parametric Compromise Solution	Fuzzy Goal Programming	Stability Set of the First Kind	Environment
Ghatee and Hashemi [31]	×	×	×	×	Fuzzy
Bustos et al. [32]	×	×	×	×	Stochastic
Alharbi et al. [30]	√	√	√	√	Fuzzy
Our proposed approach	√	√	√	√	Possibilistic

9 | Conclusion

This paper introduces a multi-objective MCF problem with possibilistic variables. A fuzzy GP approach has been applied to the possibilistic MCF problem. The advantage of this approach is the utilization to allow conflicting goals and permit the consideration of the decision environment. The GAMS software has been applied to obtain the solution. The parametric study corresponding to the α −possibly optimal solution is defined and determined without differentiability. Future work may include extending this study to other fuzzy-like structures (i. e., interval-valued fuzzy set, Neutrosophic set, Pythagorean fuzzy set, Spherical fuzzy set, etc., with more discussion and suggestive comments.

Author Contribution

H.A. K. research design, conceptualization, and validation. S. A. E. data gathering, computing, and editing. and editing. The authors have read and agreed to the published version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

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