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A New Double Frontiers Data Envelopment Analysis Approach for Assessing the Sustainability of Suppliers

Amin Zoghi*

Department of Industrial Management, Faculty of Management and Accounting, Karaj Branch, Islamic Azad University, Karaj, Iran; amin.zoghi@gmail.com.

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Abstract

One of the techniques for evaluating a supplier's sustainability is Data Envelopment Analysis (DEA). DEA is a non-parametric tool for measuring the relative efficiency of Decision-Making Units (DMUs). This paper develops a new double frontier DEA model based on the Slacks-Based Measure (SBM) for assessing suppliers' sustainability. Our proposed double frontier SBM model considers pessimistic and optimistic efficiencies. A case study is presented to demonstrate the applicability of the proposed model. The results show that the proposed model can completely rank the DMUs, and there is no tie between the overall efficiency scores.

Keywords: Double frontier, Slacks-based measure, Sustainable supplier, Data envelopment analysis, Pessimistic efficiency, Optimistic efficiency.

1 | Introduction

Incorporating sustainability in the Supply Chain Management (SCM) concept is a paradigm in which environmental, social, and economic factors are considered [1]. Carter and Rogers [2] defined Sustainable Supply Chain Management (SSCM) as "the strategic, transparent integration and achievement of an organization's social, environmental and economic goals in the systemic coordination of key inter-organizational business processes for improving the long-term economic performance of the individual and its supply chain". Generally, sustainability is defined as "using resources to meet the needs of the present without compromising the ability of future generations to meet their own needs" [3]. The broader adoption and development of sustainability can be achieved by focusing on supply chains since the supply chain considers the product from processing raw materials to delivery to the customer [4]. The problem of sustainable supplier selection requires measuring suppliers' performance with respect to economic, social, and environmental factors. Therefore, a sustainable supplier selection approach should address these factors.

Corresponding Author: amin.zoghi@gmail.com
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Recently, companies have worked with their suppliers to improve the environmental performance of their products and services [5].

One of the widely-applied tools for evaluating a supplier's sustainability is Data Envelopment Analysis (DEA). DEA is a non-parametric tool for measuring the relative efficiency of Decision-Making Units (DMUs). As Wang and Lan [6] addressed, the performance of DMUs can also be assessed by the worst efficiency frontier (pessimistic perspective). In Double Frontier Data Envelopment Analysis (DFDEA), we deal with both the best and the worst efficiency frontiers. The DFDEA approach measures two extreme operations of each DMU. As Wang et al. [7] discussed, any efficiency evaluation which considers only one of the best and the worst efficiency frontiers is biased. Furthermore, the DFDEA does not need to calculate the cross-efficiency matrix, which requires huge computations [8]. Traditional DFDEA models are based on the Charnes-Cooper-Rhodes (CCR) approach, which has two shortcomings: First, they may lead to multiple optimal solutions. Second, they can produce similar ranks that cannot discriminate against DMUs.

This paper proposes a new DEA model called Slacks-Based Measure Double Frontiers (SBMDF) to measure suppliers' sustainability. All previous works on measuring the suppliers' sustainability are based on an optimistic frontier. Our proposed SBMDF model takes into account both optimistic efficiency and pessimistic efficiency frontiers. We show that our approach can completely rank all DMUs. The main contributions of this study are as follows:

I. For the first time, we integrate the "dual form of the SBM" and the "double frontier approach".

II. We completely rank all DMUs. This breaks any tie between efficient and inefficient DMUs.

III. For the first time, we incorporate the "double frontier approach" into SSCM.

The rest of the paper is organized as follows. In Section 2, a literature review is given. In Section 3, the proposed model is presented. Section 4 provides numerical examples. A case study is given in Section 5. Finally, Section 6 concludes the paper.

2 | Literature Review

2.1 | Supplier Selection

Various mathematical programming approaches have been employed for supplier selection problems in the past, among which DEA is one of the most applied methods for supplier selection [9]. Ghodsypour and O'Brien [10] developed a mixed integer nonlinear programming model to solve multiple sourcing problems. Their model considers the total logistics cost, including net price, storage, transportation, and ordering costs. Talluri and Baker [11] proposed a multi-phase mathematical programming model for effective supply chain design. In their approach, a combination of multi-criteria efficiency models, including game theory concepts and linear and integer programming methods, were taken into account. Kumar et al. [12] applied a fuzzy goal programming model to integrate imprecise aspiration levels of the goals in a supplier selection problem. Farzipoor Saen [13] employed an Imprecise Data Envelopment Analysis (IDEA) model for supplier selection in the presence of both cardinal and ordinal data. Özgen et al. [14] integrated the Analytic Hierarchy Process (AHP) and Multiobjective Possibilistic Linear Programming (MOPLP) model for the assessment of suppliers. Wu et al. [15] proposed a fuzzy multiobjective programming model for supplier selection in the presence of various risk factors. Farzipoor Saen [16] considered the ratings for service-quality experience and service-quality credence as dual-role factors for selecting third-party reverse logistics providers.

Furthermore, Farzipoor Saen [17] proposed a method for selecting suppliers in the presence of dual-role factors and weight restrictions. The research and development cost was considered as input and output factors. Noorizadeh et al. [18] proposed a model to consider dual-role factors for supplier selection. Hosseinzadeh Zoroufchi et al. [19] developed a new cross-efficiency supplier selection model dealing with undesirable outputs. Azadi et al. [20] proposed a Chance-Constrained Data Envelopment Analysis (CCDEA) model with nondiscretionary factors and stochastic data for supplier selection. Junior et al. [21] compared fuzzy AHP and TOPSIS methods for supplier selection problems. They claimed that the fuzzy TOPSIS technique is suitable for supplier selection problems. Karsak and Dursun [22] proposed a fuzzy supplier selection methodology integrating Quality Function Deployment (QFD) and DEA. DEA has also been used in the green and SSCM context. Tavassoli et al. [23] proposed an integrated DEA model for efficiency and effectiveness assessment of suppliers.

2.2 | Sustainable Supplier Selection

Bai and Sarkis [24] integrated sustainability into supplier selection in the presence of grey systems and rough set methodologies. Amindoust et al. [25] developed a model for sustainable supplier selection in the existence of a fuzzy inference system. They determined sustainable supplier selection criteria and sub-criteria. To assess sustainable suppliers, Wen et al. [26] suggested a methodology with regard to intuitionistic fuzzy sets' group decision methods. They initially reviewed the sustainability and supplier evaluation literature and then the proposed criteria for sustainable supplier evaluation. Azadi et al. [27] proposed a new fuzzy DEA model for computing suppliers' efficiency and effectiveness in a sustainable SCM context. They extended an integrated DEA Enhanced Russell Measure (ERM) in a fuzzy context to select sustainable suppliers. Sarkis and Dhavale [28] provided a model for supplier selection for sustainable operations based on a triple-bottom-line approach employing a Bayesian framework. A triple-bottom-line (profit, people, and planet) approach was taken into account, and business operations were considered along with suppliers' environmental impacts and social responsibilities. Trapp and Sarkis [29] introduced robust portfolios of suppliers with regard to sustainability perspective. They developed an optimization model for supplier selection and supplier development.

2.3 | Double Frontiers Data Envelopment Analysis

The DFDEA is a method to overcome traditional DEA models' shortcomings. Wang et al. [7] determined the performance of DMUs via geometric average efficiency. They proposed two types of efficiency frontiers, which are optimistic and pessimistic. Wang and Chin [8] proposed an approach for selecting Advanced Manufacturing Technologies (AMT) by the DFDEA. Their model identified the best AMT. Wang and Lan [6] proposed a new model to measure Malmquist Productivity Index (MPI) using the DFDEA. The integrated MPI could show the productivity changes of DMUs over time. Ahmady et al. [30] proposed a novel fuzzy DEA model with double frontiers for supplier selection. Their proposed model was able to handle ambiguity and fuzziness in supplier selection problems. To discover the Most Productive Scale Size (MPSS), Wang and Lan [31] proposed a double frontiers approach via Hurwicz measure to incorporate both the optimistic and pessimistic frontiers. Azizi et al. [32] extended the DEADF approach in the presence of imprecise data.

However, not all the previous papers in the DFDEA can fully rank the DMUs. We show that our proposed model can fully rank the DMUs. To the best of our knowledge, there is no DFDEA model to rank DMUs fully. Moreover, there is no DFDEA model for supplier selection in the SSCM context. Therefore, this paper aims to develop a new DFDEA model for assessing suppliers' sustainability.

3 | Proposed Model

3.1 | Slacks-Based Measure

In this subsection, the SBM model is reviewed. The SBM in DEA was introduced by Tone [33]. The SBM model deals directly with input excesses and output shortfalls of DMUs. The SBM model has two important properties. First, it is "units invariant," and second, it is "monotone," decreasing in each input and output slack. Suppose we have n DMUs with the input and output matrices $X = (x_{ij}) \in R^{m\times n}$ and $Y = (y_{rj}) \in R^{p\times n}$, respectively. We suppose the data is positive, that is, X>0 and Y>0. Production Possibility Set (PPS) is defined as follows:

$$P = \{(x, y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}. \tag{1}$$

where λ is a nonnegative vector in R^n. To explain a certain DMU (xo, yo), the following expressions are considered:

$$x_o = X\lambda + s^-. \quad (2)$$

$$y_o = Y\lambda - s^+, \quad (3)$$

where $\lambda \geq 0$, $s^- \geq 0$, and $s^+ \geq 0$. The vectors $s^- \in R^m$ and $s^+ \in R^p$ represent the "input excess" and "output shortfall", respectively. From the conditions $X > 0$ and $\lambda \geq 0$, we have $xo \geq s^-$ and from $Y > 0$ and $\lambda \geq 0$ we have $y_o \geq s^+$.

To measure the efficiency of DMU_o (the DMU under evaluation), the following SBM model is formulated [33]:

$$\min\rho = \frac{1-(1/m)\sum_{i=1}^{m} s_i^-/x_{io}}{1+(1/p)\sum_{r=1}^{p} s_r^+/y_{ro}}. \quad (4)$$

s. t.

$$x_o = X\lambda + s^-. \quad (5)$$

$$y_o = Y\lambda - s^+.$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0. \quad (6)$$

Using the well-known Charnes-Cooper transformation, the nonlinear SBM model can be transformed into a linear program.

Definition 1. The DMU_o is efficient if $\varrho^*=1$.

This condition occurs when $s^{-*}=0$ and $s^{+*}=0$; i.e., there are no input excesses and no output shortfalls in any optimal solution. For an SBM inefficient DMU (x_o, y_o), we have the following expressions:

$$x_o = X\lambda^* + s^{-*}. \quad (7)$$

$$y_o = Y\lambda^* - s^{+*}. \quad (8)$$

By decreasing the input excess and increasing the output shortfall, an inefficient DMU (x_o, y_o) can be improved and become efficient as follows:

$$x_o^* \leftarrow x_o - s^{-*}. \quad (9)$$

$$y_o^* \leftarrow y_o + s^{+*}. \quad (10)$$

Using dual variables $\xi \in R$, $v \in R^m$, and $u \in R^p$, the dual formulation is as follows:

$$\max \xi \quad (11)$$

s.t.

$$\xi + vx_o - uy_o = 1,$$

$$-vX + uY \leq 0,$$

$$v \geq 1/m[1/x_o],$$

$$u \geq \xi/p[1/y_o],$$

The notation $[1/x_o]$ indicates the row vector $(1/x_{1o}, 1/x_{2o}, \ldots, 1/x_{mo})$.

3.2 | Our New SBM Model with Double Frontiers

Here, we introduce our new SBM model with double frontiers. The efficiency of $\mathrm{DMU_o}$ is defined as follows:

$$\rho=\left(\frac{1}{m}\sum_{i=1}^{m}\frac{x_{io}-s_i^-}{x_{io}}\right)\left(\frac{1}{p}\sum_{r=1}^{p}\frac{y_{ro}+s_r^+}{y_{ro}}\right)^{-1}. \tag{12}$$

Using the objective *Function (4)*, the following expression is obtained.

$$\rho=\frac{1-\frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i^-}{x_{io}}\right)}{1+\frac{1}{p}\sum_{r=1}^{p}\left(\frac{s_r^+}{y_{ro}}\right)}=\frac{\frac{x_{io}}{x_{io}}-\frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i^-}{x_{io}}\right)}{\frac{y_{ro}}{y_{ro}}+\frac{1}{p}\sum_{r=1}^{p}\left(\frac{s_r^+}{y_{ro}}\right)}=\frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{io}}{x_{io}}-\frac{s_i^-}{x_{io}}}{\frac{1}{p}\sum_{r=1}^{p}\frac{y_{ro}}{y_{ro}}+\frac{s_r^+}{y_{ro}}}=\frac{\frac{1}{m}\sum_{i=1}^{m}\frac{x_{io}-s_i^-}{x_{io}}}{\frac{1}{p}\sum_{r=1}^{p}\frac{y_{ro}+s_r^+}{y_{ro}}}. \tag{13}$$

Apparently, the *Expressions (12)* and *Expressions (13)* are equivalent. The optimistic efficiency can be extended as follows:

$$\text{Max}\,\tau_o=\theta-\varepsilon\,(s_o), \tag{14}$$

s.t.

$$\theta+v\,x_o-u\,y_o=1. \tag{15}$$

$$uY=vX-s. \tag{16}$$

$$v\geq\frac{1}{m}\left[\frac{1}{x_o}\right]. \tag{17}$$

$$u\geq\frac{\theta}{p}\left[\frac{1}{y_o}\right]. \tag{18}$$

$$\varepsilon>0,\ s\geq 0. \tag{19}$$

where s stands for the vector of slacks of DMUs, while s_o is the slack for the $\mathrm{DMU_o}$. ε is a positive non-Archimedean infinitesimal. By incorporating s_o as inefficiency in *Eq. (14)* and s in *Eq. (16)*, the discriminating power of the model is increased.

Definition 2. Optimistic Slacks-Based Measure (OSBM) efficiency.

If $s_o^*=0$ and $\tau_o^*=1$, then the $\mathrm{DMU_o}$ is OSBM-efficient; otherwise, it is referred to as non-OSBM-efficient. A linear programming *Models (14)-(19)* is solved for every DMU.

The Pessimistic Slacks-Based Measure (PSBM) of efficiency related to the $\mathrm{DMU_o}$ is measured as follows:

$$\text{Min}\,\Psi=\varphi+\varepsilon\,(s_o), \tag{20}$$

s.t.

$$\varphi+vx_o-uy_o=1. \tag{21}$$

$$vX=uY-s. \tag{22}$$

$$v\geq\frac{1}{m}\left[\frac{1}{x_o}\right]. \tag{23}$$

$$u\geq\frac{\varphi}{p}\left[\frac{1}{y_o}\right]. \tag{24}$$

$$\varepsilon>0,\ s\geq 0. \tag{25}$$

The PSBM model differs from the OSBM model. The PSBM model minimizes the efficiency of $\mathrm{DMU_o}$ while the OSBM model maximizes the efficiency of $\mathrm{DMU_o}$.

Definition 3. PSBM inefficiency.

In *Models (20)-(25)* if Ψ_o^*=1 and $s_o = 0$, the DMU_o is PSBM-inefficient; otherwise the DMU_o is non-pessimistic-inefficient.

Both optimistic and pessimistic efficiencies are used to rank efficient DMUs. The slacks are used as penalties to rank DMUs. The following expression is used for measuring the overall efficiency of DMUs for DMUs' ranking:

$$\emptyset_j^* = \frac{\tau_j^*}{\sqrt{\sum_{j=1}^n \tau_j^{*2}} + \sqrt{\sum_{j=1}^n s_j}^{\,2}} + \frac{\Psi_j^*}{\sqrt{\sum_{j=1}^n \Psi_j^{*2}} + \sqrt{\sum_{j=1}^n s_j}^{\,2}}, \quad j = 1, \dots, n. \tag{26}$$

The *Expression (26)* has the following features:

I. The slacks play penalty role in the overall efficiency measurement. If the slacks become zero, then the $\emptyset_j^*$ is increased.

II. The overall efficiency obtained from *Expression (26)* integrates both optimistic and pessimistic frontiers, which have more discriminating power.

4 | Numerical Examples

Example 1. The data set of this example is taken from Wang and Chin [8] and includes 12 Flexible Manufacturing Systems (FMS). The data set is shown in *Table 1*, in which the first input is annual capital and operating cost, and the second represents the floor space requirements of each FMS (DMU). Additionally, outputs are the improvements in qualitative benefits, Work In Process (WIP), average number of tardy jobs, and average yield.

Table 1. Data set for 12 FMSs taken from Wang and Chin [8].

	Inputs		Outputs			
FMS	Capital and Operating Cost ($100,000)	Floor Space Needed (1000ft2)	Qualitative Improvement (%)	WIP Reduced (10)	Tardiness Reduced (%)	Yield Increased (100)
1	17.02	5.0	42	45.3	14.2	30.1
2	16.46	4.5	39	40.1	13.0	29.8
3	11.76	6.0	26	39.6	13.8	24.5
4	10.52	4.0	22	36.0	11.3	25.0
5	9.50	3.8	21	34.2	12.0	20.4
6	4.79	5.4	10	20.1	5.0	16.5
7	6.21	6.2	14	26.5	7.0	19.7
8	11.12	6.0	25	35.9	9.0	24.7
9	3.67	8.0	4	17.4	0.1	18.1
10	8.93	7.0	16	34.3	6.5	20.6
11	17.74	7.1	43	45.6	14.0	31.1
12	14.85	6.2	27	38.7	13.8	25.4

All inputs and outputs are normalized. The outcome is presented in *Table 2*.

Table 2. Normalized data set for 12 FMS.

	Inputs		Outputs			
FMS	Capital and Operating Cost ($100,000)	Floor Space Needed (1000ft2)	Qualitative Improvement (%)	WIP Reduced (10)	Tardiness Reduced (%)	Yield Increased (100)
1	0.1284	0.0723	0.1453	0.1095	0.1186	0.1053
2	0.1242	0.0650	0.1349	0.0969	0.1086	0.1042
3	0.0887	0.0867	0.0900	0.0957	0.1153	0.0857
4	0.0794	0.0578	0.0761	0.0870	0.0944	0.0874
5	0.0717	0.0549	0.0727	0.0827	0.1003	0.0714
6	0.0361	0.078	0.0346	0.0486	0.0418	0.0577

Table 2. Continued.

FMS	Inputs: Capital and Operating Cost ($100,000)	Inputs: Floor Space Needed (1000ft2)	Outputs: Qualitative Improvement (%)	Outputs: WIP Reduced (10)	Outputs: Tardiness Reduced (%)	Outputs: Yield Increased (100)
7	0.0468	0.0896	0.0484	0.0641	0.0585	0.0689
8	0.0839	0.0867	0.0865	0.0868	0.0752	0.0864
9	0.0277	0.1156	0.0138	0.0421	0.0008	0.0633
10	0.0674	0.1012	0.0554	0.0829	0.0543	0.0721
11	0.1338	0.1026	0.1488	0.1102	0.1170	0.1088
12	0.1120	0.0896	0.0934	0.0935	0.1153	0.0888

After running the traditional double frontiers CCR approach [8], the optimistic, pessimistic, and overall efficiencies of each FMS are calculated. Results are depicted in *Table 3*. Furthermore, FMSs are ranked according to their overall efficiency scores. According to *Table 3*, FMS #2 and #9 obtain the same overall efficiency score equal to 0.5631. Thus, both of them are ranked as the eighth FMS. This fact reveals that the traditional DFDEA approach cannot rank the DMUs.

Table 3. Results of the traditional double frontiers CCR approach [8].

FMS	Optimistic Efficiency	Pessimistic Efficiency	Overall Efficiency	Rank
1	1	1.0146	0.5670	7
2	1	1	0.5631	8
3	0.9824	1.1193	0.5898	5
4	1	1.1921	0.6144	2
5	1	1.2227	0.6226	1
6	1	1.1515	0.6036	4
7	1	1.1587	0.6055	3
8	0.9614	1.0748	0.5717	6
9	1	1	0.5631	8
10	0.9536	1	0.5494	11
11	0.9831	1	0.5581	10
12	0.8012	1	0.5043	12

Now, our SBMDF approach is run for the data set. The ε is set to 0.001. The results of the proposed SBMDF model are depicted in *Table 4*.

Table 4. Results of the proposed SBMDF model.

FMS	Optimistic Efficiency	Pessimistic Efficiency	Overall Efficiency	Rank
1	1	1.2645	0.4520	4
2	1	1	0.3328	10
3	0.8543	1.3318	0.4479	5
4	1	1.8080	0.5235	1
5	1	1.6425	0.4927	2
6	1	1.4765	0.3501	9
7	1	1.5030	0.4676	3
8	0.8066	1.2596	0.4024	6
9	1	1	0.0168	12
10	0.7117	1	0.2892	11
11	0.8294	1	0.3899	7
12	0.7507	1	0.3728	8

Table 4 reveals that our model can rank all DMUs. However, using Wang and Chin's [8] model, the FMSs #2 and #9 have identical ranks while employing our proposed model, the FMSs #2 and #9 have the rank 10 and 12, respectively.

Spearman's correlation coefficient analysis determines the correlation coefficient between the CCR double frontiers model and our proposed approach. *Table 5* shows the outcome. Given *Table 5*, the correlation coefficient between the results of the two approaches is significant (0.848) at the level of 0.01.

Table 5. The correlation coefficient between the CCR double frontiers approach and the SBMDF approach.

			CCR Double Frontiers	SBMDF
Spearman's rho	CCR double frontiers	Correlation	1.000	0.848**
		Sig. (2-tailed)	0	0.000
	SBMDF	Correlation	0.848**	1.000
		Sig. (2-tailed)	0.000	0

** Correlation is significant at the 0.01 level (2-tailed).

Example 2. The data set of this example is taken from Wang et al. [7], which is shown in *Table 6*. There are 10 DMUs with one input and two outputs. All inputs and outputs are normalized and are shown in *Table 7*.

Table 6. Data set taken from Wang et al. [7].

DMUs	Input (X_1)	Output 1 (Y_1)	Output 2 (Y_2)
1	1	1	8
2	1	2	3
3	1	2	6
4	1	3	3
5	1	3	7
6	1	4	2
7	1	4	5
8	1	5	2
9	1	6	2
10	1	7	1

Table 7. Normalized data set.

DMUs	Input (X_1)	Output 1 (Y_1)	Output 2 (Y_2)
1	0.1	0.027	0.2051
2	0.1	0.0541	0.0769
3	0.1	0.0541	0.1538
4	0.1	0.0811	0.0769
5	0.1	0.0811	0.1759
6	0.1	0.1081	0.0513
7	0.1	0.1081	0.1282
8	0.1	0.1351	0.0513
9	0.1	0.1622	0.0513
10	0.1	0.1892	0.0256

Table 8 presents the results provided by Wang et al. [7]. The third and fourth columns represent the optimistic and pessimistic efficiencies, respectively. The fifth column shows the geometric average efficiencies of DMUs. The sixth column depicts the ranks of DMUs. As is seen in *Table 8*, the DMUs #1 and #10 have similar ranks. This shows that the CCR-double frontiers approach proposed by Wang et al. [7] cannot rank the DMUs.

Table 8. Ranking of DMUs obtained from the CCR-double frontiers [7].

DMUs	Optimistic Efficiency	Pessimistic Efficiency	Geometric Average Efficiency	Rank
1	1	1	1.0000	5
2	0.5217	1	0.7223	10
3	0.8235	1.2308	1.0068	4
4	0.6522	1.1250	0.8566	8
5	1	1.6923	1.3009	1
6	0.6957	1	0.8341	9

Table 8. Continued.

DMUs	Optimistic Efficiency	Pessimistic Efficiency	Geometric Average Efficiency	Rank
7	0.9565	1.7500	1.2938	2
8	0.8261	1.1000	0.9533	7
9	0.9565	1.2000	1.0714	3
10	1	1	1.0000	5

To solve this difficulty, we run our proposed SBMDF models. The results are shown in *Table 9*. As is seen, the SBMDF can successfully rank all DMUs. According to the *Table 9*, the overall efficiency of the DMU #7 is 0.5885. Thus, it is selected as the best DMU. Conversely, the overall efficiency of DMU #1 is 0.2103, known as the worst DMU. As can be seen, there is no tie between overall efficiencies. This implies the high discrimination power of our proposed approach.

Table 9. Results of our proposed SBMDF model.

DMUs	Optimistic efficiency	Pessimistic efficiency	Overall efficiency	Rank
1	1	1	0.2103	10
2	0.5237	1	0.3225	7
3	0.7158	1.3336	0.4030	3
4	0.6079	1.1999	0.3803	5
5	1	1.8119	0.5697	2
6	0.5405	1	0.2872	8
7	0.9618	1.8184	0.5885	1
8	0.6741	1.1111	0.3342	6
9	0.8964	1.2008	0.3875	4
10	1	1	0.2610	9

Table 10 shows Spearman's correlation coefficient analysis between the CCR double frontiers and our proposed SBMDF approach. The correlation coefficient between the results of the two approaches is significant (0.657) at the level of 0.05. This result confirms that the proposed model yields a valid result.

Table 10. The correlation coefficient between the proposed approach and the CCR double frontiers approach.

			CCR Double Frontiers	SBMDF
Spearman's rho	CCR double frontiers	Correlation coefficient	1.000	0.657*
		Sig. (2-tailed)	0	0.039
	SBMDF	Correlation coefficient	0.657*	1.000
		Sig. (2-tailed)	0.039	0

5 | Case Study

A case study is presented in this section to demonstrate the applicability of our proposed approach. Iranian Distribution Companies Association (IDCA) seeks to preserve, protect, and develop the distribution industry. IDCA wishes to assess the sustainability of suppliers. This case study evaluates sixteen Iranian paint and resin suppliers (DMUs). The companies produce paints, inks, and adhesives to maintain, protect, and decorate products.

As mentioned, sustainability refers to considering social, environmental, and economic factors [2]. Given this definition, the criteria are selected to measure suppliers' sustainability. The data set dates back to 2012, as depicted in *Table 11*. The DMUs have three inputs and two outputs. The inputs include annual cost, energy cost, and annual environmental cost. The first output is the number of trained personnel in job, safety, and health. The second output is total profit. Annual cost and total profit are economic factors. Energy cost and annual environmental cost are environmental factors. Besides, number of trained personnel is a social factor. Other scholars have used these factors. For instance, Khodakarami et al. [34] used energy and annual environmental costs to measure the sustainability of 32 industrial parks. Gheidari-Kheljani et al. [35] considered annual cost as an input for solving supplier selection problems. Amindoust et al. [25] took into account profit as an output for ranking sustainable suppliers based on a fuzzy inference system. Hashemi et al. [36] integrated a green supplier selection approach and Analytic Network Process (ANP) technique in which several trained personnel were used as an output.

Table 11. Data set.

	DMUs	Inputs			Outputs	
		Annual Cost ($)	Energy Cost ($)	Annual Environmental Cost ($)	Number of Trained Personnel	Total Profit ($)
1	Aria Resin Co.	994000	38116	39000	158	1586667
2	Azar Resin Co.	894667	95289	33667	191	1080000
3	Peka Chemie Co.	1251000	28587	28000	217	1616667
4	Bonyan Kala Chemie Co.	987000	19058	40333	295	1396667
5	Pars Pamchal Chemical Co.	929667	66702	38667	337	1570000
6	Paint Sahar Co.	983667	114347	45000	263	1503333
7	Taba Coatings	952000	38116	58000	338	1643333
8	Paksan Co.	884667	85760	44000	194	1450000
9	Chemical Carbon Acid Co.	973667	38116	36667	172	1376667
10	Alborz Chelic Co.	907667	133405	32667	387	1286667
11	Mobin Petrochemical Co.	1325000	95289	54667	419	1719000
12	Marun Petrochemical Co.	618333	123876	45000	476	1410000
13	Fajr Petrochemical Co.	1395333	57174	46333	117	1990000
14	Laleh Petrochemical Co.	924667	38116	37333	218	1123333
15	Khosh & Kcc Co.	885667	85760	58667	176	1556667
16	Rang Afarin Co.	1284000	95289	53667	197	1703333

In *Table 12*, the data set is normalized.

Table 12. Normalized data set.

	Dmus	Inputs			Outputs	
		Annual Cost	Energy Cost	Annual Environmental Cost	Number of Trained Personnel	Total Profit
1	Aria Resin Co.	0/0614	0/0331	0/0564	0/0380	0/0661
2	Azar Resin Co.	0/0553	0/0826	0/0487	0/0460	0/0450
3	Peka Chemie Co.	0/0773	0/0248	0/0405	0/0522	0/0673
4	Bonyan Kala Chemie Co.	0/0610	0/0165	0/0583	0/0710	0.0582
5	Pars Pamchal Chemical Co.	0/0574	0/0579	0/0559	0/0811	0/0654
6	Paint Sahar Co.	0/0608	0/0992	0/0651	0/0633	0/0626
7	Taba Coatings	0/0588	0/0331	0/0839	0/0813	0/0684
8	Paksan Co.	0/0546	0/0744	0/0636	0/0467	0/0604
9	Chemical Carbon Acid Co.	0/0601	0/0331	0/0530	0/0414	0/0573
10	Alborz Chelic Co.	0/0561	0/1157	0/0472	0/0931	0/0536
11	Mobin Petrochemical Co.	0/0818	0/0826	0/0790	0/1008	0/0716
12	Marun Petrochemical Co.	0/0382	0/1074	0/0651	0/1146	0/0587

Table 12. Continued.

	Dmus	Inputs			Outputs	
		Annual Cost	Energy Cost	Annual Environmental Cost	Number of Trained Personnel	Total Profit
13	Fajr Petrochemical Co.	0/0862	0/0496	0/0670	0/0282	0/0829
14	Laleh Petrochemical Co.	0/0571	0/0331	0/0540	0/0525	0/0468
15	Khosh & Kcc Co.	0/0547	0/0744	0/0848	0/0424	0/0648
16	Rang Afarin Co.	0/0793	0/0826	0/0776	0/0474	0/0709

The results are depicted in *Table 13*.

Table 13. Evaluation of suppliers' sustainability by the SBMDF model.

DMUs		Optimistic Efficiency	Pessimistic Efficiency	Overall Efficiency	Rank
1	Aria Resin Co.	1	1.4556	0.2501	13
2	Azar Resin Co.	0.5846	1	0.2592	9
3	Peka Chemie Co.	1	2.1438	0.3548	4
4	Bonyan Kala Chemie Co.	1	3.1678	0.4072	2
5	Pars Pamchal Chemical Co.	1	1.7327	0.4211	1
6	Paint Sahar Co.	0.6636	1.1511	0.3089	6
7	Taba Coatings	1	2.0184	0.3842	3
8	Paksan Co.	0.6097	1.1212	0.2755	7
9	Chemical Carbon Acid Co.	0.6720	1.4798	0.2735	8
10	Alborz Chelic Co.	1	1	0.2584	10
11	Mobin Petrochemical Co.	0.7568	1.4028	0.3508	5
12	Marun Petrochemical Co.	1	1	0.2562	11
13	Fajr Petrochemical Co.	0.4551	1	0.2044	15
14	Laleh Petrochemical Co.	0.6793	1	0.1693	16
15	Khosh & Kcc Co.	0.5318	1	0.2454	14
16	Rang Afarin Co.	0.5309	1	0.2525	12

Table 13 shows that Aria Resin, Peka Chemie, Bonyan Kala Chemie, Pars Pamchal Chemical, Taba Coatings, Alborz Chelic, and Marun Petrochemical are optimistically efficient, while the rest of the DMUs are inefficient. The fourth column represents the pessimistic efficiency scores, of which Azar Resin, Alborz Chelic, Marun Petrochemical, Fajr Petrochemical, Laleh Petrochemical, Khosh and KCC, and Rang Afarin are pessimistically inefficient. The fifth column represents the overall efficiency scores obtained from the *Expression (26)*. The last column of *Table 13* shows the rank of DMUs in terms of overall efficiency scores. It is clear that there is no tie in the ranking of DMUs. Pars Pamchal Chemical Co. is the most sustainable supplier, and Laleh Petrochemical Co. is the worst sustainable supplier.

6 | Conclusion

One of the essential subjects of DEA studies is the ranking of DMUs. Decision-makers need to observe their firms' rank within the industry. Thus, a complete ranking of DMUs has recently been the center of studies. There have been traditional DEA methods regarding full ranking, such as cross-efficiency, super-efficiency technique, evaluation of DMUs through benchmarking, canonical correlation analysis, and discriminant analysis. The cross-efficiency method computes the efficiency score of each DMU n times, employing the optimal weights measured by the n linear programs. The supper-efficiency technique brings this opportunity for DMU k to achieve an efficiency score greater than one by removing the kth constraint in the primal solution. The benchmarking method ranks the efficient DMUs by computing their importance as a benchmark for inefficient DMUs.

In this method, the additive model is first used to evaluate the score of slacks, and then another DEA model is applied to all DMUs to rank them fully. Canonical correlation is an expanded form of regression analysis. Canonical correlation analyses multiple inputs and multiple outputs to search for a single vector weight for the inputs and outputs. The discriminating analysis method can be divided into two groups: linear discriminating analysis for ranking and second discriminating analysis of ratios for ranking. Prior, linear discriminating analysis was used to find a score function that ranks DMUs based on efficiency and inefficiency. Instead of considering a linear combination of inputs and outputs in one equation, a ratio function is constructed between a linear combination of inputs and a linear combination of outputs.

To sum up, as can be easily seen, all the methods mentioned have a weak point in including too many calculations. Likewise, dividing the DMUs into efficient and inefficient at the first stage is an inseparable part of those methods, implying more calculations and complexity. The DFDEA approach has been developed to deal with the best and worst efficiency frontiers. Although the number of calculations has decreased in the DFDEA approach, it has the limitation of having low discriminating power for the ranking of DMUs. For instance, it yields two DMUs with similar ranking scores. Therefore, it cannot be employed to rank DMUs completely. However, based on the proposed model in this article, all DMUs are ranked in just one step without the same ranking for two different DMUs.

Sustainable supplier selection is vital for many successful companies. Therefore, a significant body of our case study has been conducted to assess suppliers' sustainability. A new model was proposed to rank suppliers in terms of sustainability. The case study showed that the proposed SBMDF model broke the tie in DMUs' ranking. Furthermore, it proved that the discriminating power of the proposed model is more than that of previous DFDEA approaches. Using both optimistic and pessimistic frontiers, our SBMDF model provided a comprehensive viewpoint for managers to select the best supplier. Based upon the results of the case study, first, by discovering the sustainability scores of DMUs, the more efficient DMUs can be selected as benchmarks for the less efficient ones. Second, when comparative performance information is available, it turns out to be a means for the self-motivation of sustainable companies. Finally, stakeholders such as NGOs are informed of suppliers' sustainable performance. Hence, sustainable suppliers enjoy national and international reputations for their sustainable operations.

Further research can be done, some of which are as follows: Similar research can be repeated in the presence of dual-role factors. This study used the proposed model to assess the sustainability of suppliers. Another research topic is to apply our proposed models to other problems such as technology selection, market selection, etc.

Author Contributions

Amin Zoghi designed the study, developed the new Double Frontiers DEA approach, and conducted the data analysis. He also interpreted the results and prepared the manuscript.

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Data Availability

The data used to assess the sustainability of suppliers in this study are proprietary and cannot be publicly disclosed. However, summarized data and methodological details can be shared upon reasonable request to the corresponding author.

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