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An EOQ Model for Two Warehouse System During Lock-Down Considering Linear Time Dependent Demand

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Abstract

The COVID-19 epidemic had a significant impact on both India and the rest of the world. The manufacturing and selling processes have been delayed due to the Covid-19 epidemic's quick spread. Many sectors are now searching for a suitable and efficient disruption recovery strategy to assist in their recovery. Thus, the goal of this essay is to create a workable model that takes the Covid-19 pandemic's many elements into account. This study proposes an inventory model while taking into account an interruption in demand. In this model, the FIFO policy is used to analyze effect of degradation. It has been suggested that a two-parametric Weibull distribution would accurately reflect the actual issues brought on by degradation. A two-warehouse system's total cost will be as low as possible during the lock-down period, according to the research. Additionally, sensitivity analysis was utilized to assess the behavior of the models.

Keywords: Demand disruptions, Perishables items, EOQ model, Covid-19 lock-down, Deterioration, Linear demand.

1 | Introduction

Along with the worldwide tragedy of human fatalities, Covid-19 has an impact on other economic sectors and activities, including manufacturing, supply chain logistics, etc. [1]. Small cities in low-income nations like India, Pakistan, Afghanistan, Bangladesh, Nepal, Bhutan, Thailand, Myanmar, and the Maldives are experiencing financial difficulties as a result of the lock-down [2] and closure imposed by their respective governments. The government closes the borders of many cities for security reasons. Therefore, in order to maintain their company, small and medium-sized business owners require the most cost-effective EOQ

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inventory design for products. A quick increase in demand coupled with a lack of significant raw materials because of disturbances in the global supply chain is the worst-case scenario Covid-19 has brought about in terms of vital stock availability [3]. The creation and distribution of vaccines may potentially be hampered by such a supply chain network breakdown [4].

The rate of perishable product degradation has associated effects. Because a perishable product's rate of degradation is closely tied to its quality, it affects customer demand in a similar way. For instance, freshness is a crucial aspect of the quality of an Agri-fresh product. The look of an Agri-fresh product is often used as a proxy by customers to assess its freshness [5]. Bhunia and Shaikh [6] looked at a predictive inventory model for things that continually degrade. Dye and Yang [7] looked at how preservation investments affected the optimum course of action for an inventory model for perishable goods. According to [8], heterogeneous product quality with a rate of degradation at various temperatures and stages is taken into account. Tiwari et al. [9] provided a supply chain inventory model with an expiry date to ascertain the ideal pricing and replenishment cycle. For perishable items, Shaikh et al. [10] developed an EOQ method in which degradation starts after a certain length of time has passed since the goods were kept. Yang et al. [11] research focuses on controlling degradation rate by treating degradation as a constant. Garg et al. [12] created a methodology to deal with a two-warehouse inventory model for perishable goods with partial backlog in an effort to increase total profit. Rana et al. [13] built research for two warehouse systems for products that were going bad while taking demand disruption during the Covid epidemic into account. Kumar et al. [14] investigated a model for slowly decaying goods in a two-warehouse system under inflationary circumstances. Das et al. [15] takes into account a two-warehouse system with a demand that varies linearly with time and a two-parameter Weibull distribution that accounts for degradation.

The demand function is crucial in developing strategies for inventory models. It varies according on several factors, including time, quality, special offers, etc. The selling price, supply level, product greenness, timing, and other variables all have an impact on variable demand. Using a time- and price-dependent demand rate, Maihami and Kamalabadi [16] examined a joint pricing inventory model. The multi-period inventory routing problem was dealt with by [17] using stochastic demand. Recent publications in this area include works by [18]–[21] and others. The major aspects of our contribution are compared to published papers in *Table 1*.

Rana et al. [13] investigated an EOQ model for a two-warehouse system that incorporates linear time-dependent demand while taking Covid-19 lock-down demand disruption into account. The article takes two scenarios, one for a short lock-down time and the other for a lengthy lock-down period, to describe the Covid-19 pandemic issue. Rana et al. [13] does not consider the possibility that a government may be late in declaring lock-down in order to control the Covid-19 transmission rate. This research examines the prospect of an unavoidable scenario similar to Covid-19, in which the government may be late in declaring a lock-down or shut-down, and result in the inventory of Owned Warehouse (OW) being fully depleted before the lock-down start owing to customer over stocking. Additionally, to add realism to the model, Weibull distribution is used to account for the pace of degradation. This model takes the First In, First Out (FIFO) method into account to lower the effect of deterioration.

The remaining portions of the study explains follows; in Section 2, the research classify problem description, mathematical notation and assumption. The required solution procedure and a numerical example taken to validate the research model in Section 3. In Section 4 required sensitivity analysis processed for various parameter. Finally, a justified conclusion draws to the research model in Section 5.

Table 1. Comparing our suggested research against previously published research.

Ref.	Demand Function	Degradation Function	Degradation Rate	Demand Disruption	Objective Function
[22]	Constant	Constant	Instantaneous	No	Cost
[23]	Exponentially vary with time	Constant	Instantaneous	No	Cost
[24]	Increase over Time	Constant	Instantaneous	No	Profit
[25]	Based on price	Constant	Non-instantaneous	No	Profit
[26]	Based on price	Constant	Instantaneous	No	Cost
[27]	Stoke and time dependent	Two Parameter Weibull distribution	Instantaneous	No	Cost
[28]	Based on price	Constant	Instantaneous	No	Profit
[9]	Based on price	Based on time	Instantaneous	No	Profit
[10]	Based on price	Three parameter Weibull distribution	Non-instantaneous	No	Profit
[29]	Based on stocks	Constant	Non-instantaneous	No	Cost
[30]	Inflation induced time dependent	Constant	Instantaneous	No	Cost
[31]	Based on Backlogged	Constant	Instantaneous	No	Cost
[14]	Based on Stocks	Constant	Instantaneous	No	Cost
Proposed Project	Time dependent	Two parameter Weibull distribution	Instantaneous	Yes	Cost

2 | Mathematical Model, Assumptions, and Notations

2.1 | Assumption

I. Two-parametric Weibull distribution is taken into account for degradation rate.

II. The lead time has been supposed insignificant.

III. We can start keeping goods in a Rented Warehouse (RW) when an OW completely full.

IV. The OW has a finite amount of storage capacity for W units, whereas the RW has an infinite amount of storage space.

V. The inventory management tools can handle single kind of product.

VI. The damaged product cannot be fixed.

VII. The government's imposition of the entire lock-down and subsequent lifting is referred to as periods T_L and T_0, respectively.

VIII. The preliminary demand rate (α) is assumed to represent a percentage of Δd and Δd_1.

IX. The FIFO dispatched technique is applied when the OW wipes out of capacity, which means that the first product in the OW consumed before the last product in the RW.

2.2 | Notations

Table 2 presents the key notations used in this work, aimed at enhancing clarity.

Table 2. Notations of the work.

Notation	Description
$D(t)$	The demand rate at time t, we assumed $D(t) = \alpha + \beta t$ where α is the starting demand rate, and β is the rate at which demand rises over time.
T	Total time period.
T_L	The period of time when the government-imposed lock-down.
T_O	The point at which the government has relaxed the lock-down.
t_w	When OW is completely empty.
Z	Lock-down period (i.e., $T_O - T_L$).
C_P	Purchase price per stock.
G	Holding costs per stock per unit time in OW.
E	Holding costs per stock per unit time in RW.
Δd	Definite drop-in demand rate due to lock-down Limitations.
Δd_1	Definite rise in demand rate due to lock-down Limitations relaxed by government.
$I_{o1}(t)$	Inventory quantity at time t in OW in time intervals 0 toT_L.
$I_{r1}(t)$	Inventory quantity at any given moment t in RW at various intervals, where $i = 1,2,3,4$.
Q	Total order quantity.
W	Storage of OW.
X	Stocks to be store in RW (i.e., $Q - W$).
a	The OW and RW degradation rate's scale parameter.
b	The OW and RW degradation rate's shape parameter.

2.3 | Mathematical Model Development

First off, the merchant purchased Q quantities of items, which is more than OW could handle. The shop must thus store the excess inventory in a RW. Since the holding cost of RW is higher than OW owing to having advanced storage facilities, the shop first stored things in OW, or W. After completing OW, the merchant stores the leftover products $X = Q - W$ in RW. We employ FIFO policy, which means that after entirely evacuating OW, the merchant uses RW's stock, in order to reduce the effect of degradation. The inventories of OW diminish from 0 to t_w due to a combination of demand and degradation, and at t_w, the OW is eventually fully empty. The RW only saw degradation throughout this time. Following t_w, the RW begins to diminish as a result of demand and degradation, eventually falling to zero at T. *Fig. 1* shows all of the important facts described previously.

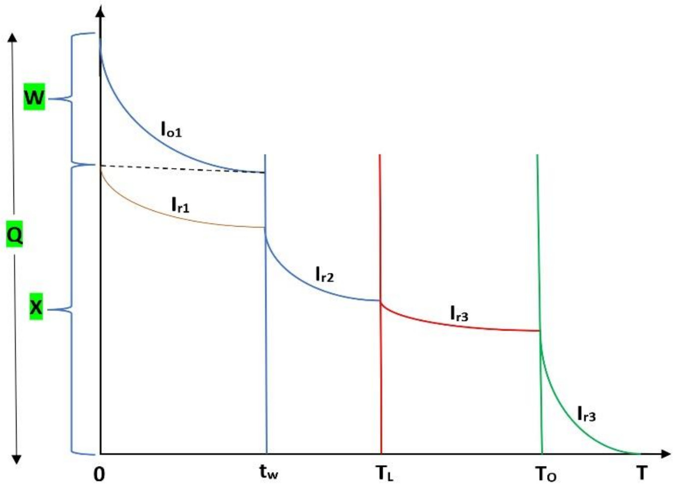


Fig. 1. Graphical illustration.

During the time $0 \to t_w$, the inventory at the own warehouse decreases due to effect of the both degradation and demand, which is represented by the differential *Eq. (1)*,

$$\frac{dI_{o1}(t)}{dt} + abt^{b-1}I_{o1}(t) = -(\alpha + \beta t). \tag{1}$$

Using the boundary condition $I_{o1}(0) = W$, the solution of the above differential equation is:

$$I_{o1}(t) = e^{-at^b}\left[W - t\left(\alpha + \frac{t\beta}{2}\right) - at^{b+1}\left(\frac{\alpha}{b+1} + \frac{\beta t}{b+2}\right)\right]. \tag{2}$$

Calculate t_w (the time at which OW completely empty)

Allow the OW is to be ultimately fully emptied at time t_w i.e., $I_{o1}(t_w) = 0$, where $0 \le t_w \le T_L$.

Now $I_{o1}(t_w) = 0$.

$$W - t_w\left(\alpha - \frac{t_w\beta}{2}\right) - at_w^{b+1}\left(\frac{\alpha}{b+1} + \frac{\beta t_w}{b+2}\right) = 0. \tag{3}$$

Neglecting the higher power of t_w i.e., t_w^{b+1} and t_w^{b+2} from *Eq. (3)*, we get

$$t_w = \frac{-\alpha \pm \sqrt{\alpha^2 + 2\beta W}}{\beta}. \tag{4}$$

Throughout the time period 0 to t_w the inventory in RW, I_{r1}, face only degradation, So, demand will be zero, therefore inventory of I_{r1} represented by *Eq. (5)*,

$$\frac{dI_{r1}(t)}{dt} + abt^{b-1}I_{r1}(t) = 0. \tag{5}$$

Using the initial condition $I_{r1}(0) = X$, the above differential equations answer is:

$$I_{r1}(t) = Xe^{-at^b}. \tag{6}$$

Throughout the time period t_w to T_L, the inventory in RW, $I_{r2}(t)$, face both degradation and demand $(\alpha + \beta t)$, therefore

$$\frac{dI_{r2}(t)}{dt} + abt^{b-1}I_{r2}(t) = -(\alpha + \beta t). \tag{7}$$

Using initial condition $I_{r1}(t_w) = I_{r2}(t_w)$, the above differential equation's answer is:

$$I_{r2}(t) = e^{-at^b}\left[\alpha(t_w - t) + \frac{\beta}{2}(t_w^2 - t^2) + \frac{\alpha a}{b+1}\left(t_w^{b+1} - t^{b-1}\right) + \frac{\beta a}{b+2}\left(t_w^{b+2} - t^{b+2}\right) + X\right]. \tag{8}$$

During $(T_L \le t \le T_O)$ the inventory in RW, $I_{r3}(t)$, face a decrease in demand rate (Δd), and represented by differential *Eq. (8)*:

$$\frac{dI_{r3}(t)}{dt} + abt^{b-1}I_{r3}(t) = -(\alpha + \beta t - \Delta d). \tag{9}$$

Using initial condition $I_{r2}(T_L) = I_{r3}(T_L)$, the above differential equations answer is:

$$I_{r3}(t) = e^{-at^b}\left[X + (\Delta d - \alpha)\left(t - T_L + \frac{a}{b+1}\left(t^{b+1} - T_L^{b+1}\right)\right) - \beta\left(\frac{1}{2}\left(t^2 - T_L^2\right) + \frac{a}{b+2}\left(t^{b+2} - T_L^{b+2}\right)\right)\right]. \tag{10}$$

During $(T_O \le t \le T)$, The inventory in RW, $I_{r4}(t)$, face a increase in demand rate (Δd_1), and represented by differential *Eq. (11)*.

$$\frac{dI_{r4}(t)}{dt} + abt^{b-1}I_{r4}(t) = -(\alpha + \beta t + \Delta d_1). \tag{11}$$

Using initial condition $I_{r4}(T) = 0$, the above differential equations answer is:

$$I_{r4}(t) = e^{-at^b}\left[(\alpha + \Delta d_1)\left((T - t) + \frac{a}{b+1}\left(T^{b+1} - t^{b+1}\right)\right) + \beta\left(\frac{1}{2}(T^2 - t^2) + \frac{a}{b+2}\left(T^{b+2} - t^{b+2}\right)\right)\right]. \quad \textbf{(12)}$$

Cost evaluation

I. The ordering costs is OC.

II. Total inventory holding costs in (C_H).

$$C_H = G\left[\int_0^{t_w} I_{o1}(t)dt\right] + E\left[\int_0^{t_w} I_{r1}(t)dt + \int_{t_w}^{T_L} I_{r2}(t)dt + \int_{T_L}^{T_O} I_{r3}(t)dt + \int_{T_O}^{T} I_{r4}(t)dt\right], \quad \textbf{(13)}$$

$$\begin{aligned} C_H = {} & G\left[\int_0^{t} e^{-at^b}\left[W - t\left(\alpha + \frac{\beta t}{2}\right) - at^{b+1}\left(\frac{\alpha}{b+1} + \frac{\beta t}{b+2}\right)\right] dt\right] \\ & + E\left[\int_0^{t_w} X\, e^{-at^b} dt\right. \\ & + \int_{t_w}^{T_L} e^{-at^b}\left[\alpha(t_w - t) + \frac{\beta^2}{2}(t_w^2 - t) + \frac{\alpha a}{b+1}\left(t_w^{b+1} - t^{b+1}\right) + \frac{\beta a}{b+2}\left(t_w^{b+2} - t^{b+2} + \right.\right. \\ & + \int_{T_L}^{T_O} e^{-at^b}\left[X + (\Delta d - \alpha)\left(t - T_L + \frac{a}{b+1}\left(t^{b+1} - T_L^{b+1}\right)\right)\right. \\ & \left. - \beta\left(\frac{1}{2}(t^2 - T_L^2) + \frac{a}{b+2}\left(t^{b+2} - T_L^{b+2}\right)\right)\right] dt \\ & + \int_{T_O}^{T} e^{-at^b}\left[(\alpha + \Delta d_1)\left((T - t) + \frac{a}{b+1}\left(T^{b+1} - t^{b+1}\right)\right)\right. \\ & \left.\left. + \beta\left(\frac{1}{2}(T^2 - t^2) + \frac{a}{b+2}\left(T^{b+2} - t^{b+2}\right)\right)\right] dt\right]. \end{aligned}$$

III. Purchasing costs per cycle is C_pQ.

IV. Total degradation cost is (C_D).

$$C_D = C_P\left[\int_0^{t_w} abt^{b-1} I_{O1}(t)dt + \int_0^{t_w} ab\, t^{b-1} I_{r1}(t)dt + \int_{t_w}^{T_L} ab\, t^{b-1} I_{r2}(t)dt + \int_{T_L}^{T_O} ab\, t^{b-1} I_{r3}(t)dt + \int_{T_O}^{T} ab\, t^{b-1} I_{r4}(t)dt\right]. \quad \textbf{(14)}$$

$$\begin{aligned} C_D = {} & C_p\left[\int_0^{t_w} ab\, t^{b-1} e^{-at^{b-1}}\left[W - t\left(\alpha + \frac{\beta t}{2}\right) - at^{b+1}\left(\frac{\alpha}{b+1} + \frac{\beta t}{b+2}\right)\right] dt + \int_0^{t_w} ab\, t^{b-1} X e^{-at^b} dt\right. \\ & + \int_{t_w}^{T_L} ab\, t^{b-1} e^{-at^b}\left[\alpha(t_w - t) + \frac{\beta}{2}(t_w^2 - t^2) + \frac{\alpha a}{b+1}\left(t_w^{b+1} - t^{b-1}\right)\right. \\ & \left. + \frac{\beta a}{b+2}\left(t_w^{b+2} - t^{b+2}\right) + X\right] dt \\ & + \int_{T_L}^{T_O} ab\, t^{b-1} e^{-at^b}\left[X + (\Delta d - \alpha)\left(t - T_L + \frac{a}{b+1}\left(t^{b+1} - T_L^{b+1}\right)\right)\right. \\ & \left. - \beta\left(\frac{1}{2}(t^2 - T_L^2) + \frac{a}{b+2}\left(t^{b+2} - T_L^{b+2}\right)\right)\right] dt \\ & + \int_{T_O}^{T} ab\, t^{b-1} e^{-at^b}\left[(\alpha + \Delta d_1)\left((T - t) + \frac{a}{b+1}\left(T^{b+1} - t^{b+1}\right)\right)\right. \\ & \left.\left. + \beta\left(\frac{1}{2}(T^2 - t^2) + \frac{a}{b+2}\left(T^{b+2} - t^{b+2}\right)\right)\right] dt\right]. \end{aligned}$$

$$
\begin{aligned}
C_H = G\Bigg[& W\left(t_w - \frac{at_w^{b+1}}{b+1}\right) - \alpha t_w^2\left(\frac{1}{2} - \frac{at_w^b}{b+2}\right) - \frac{\alpha a t_w^{b+2}}{b+1}\left(\frac{1}{b+2} - \frac{at_w^b}{2b+2}\right) - \frac{\beta t_w^3}{2}\left(\frac{1}{3} - \frac{at_w^b}{b+3}\right) \\
& - \frac{a\beta t_w^{b+3}}{b+2}\left(\frac{1}{b+3} - \frac{at_w^b}{2b+3}\right)\Bigg] \\
& + E\Bigg[X\left(t_w - \frac{at_w^{b+1}}{b+1}\right) \\
& + \alpha\left[t_w(T_L - t_w) - \frac{1}{2}\left(T_L^2 - t_w^2\right) - \frac{at_w}{b+1}\left(T_L^{b+1} - t_w^{b+1}\right) \right. \\
& \left. + \frac{a}{b+1}\left(T_L^{b+2} - t_w^{b+2}\right)\right] \\
& + \frac{\beta}{2}\left[t_w^2(T_L - t_w) - \frac{1}{3}\left(T_L^3 - t_w^3\right) - \frac{at_w^2}{b+1}\left(T_L^{b+1} - t_w^{b+1}\right) \right. \\
& \left. + \frac{a}{b+3}\left(T_L^{b+3} - t_w^{b+3}\right)\right] \\
& + \frac{a\alpha}{b+1}\left[t_w^{b+1}(T_L - t_w) - \frac{1}{b+2}\left(T_L^{b+2} - t_w^{b+2}\right) - \frac{at_w^{b+1}}{b+1}\left(T_L^{b+1} - t_w^{b+1}\right) \right. \\
& \left. + \frac{a}{2b+2}\left(T_L^{2b+2} - t_w^{2b+2}\right)\right] \\
& + \frac{a\beta}{b+2}\left[t_w^{b+2}(T_L - t_w) - \frac{1}{b+3}\left(T_L^{b+3} - t_w^{b+3}\right) - \frac{at_w^{b+2}}{b+2}\left(T_L^{b+1} - t_w^{b+1}\right) \right. \\
& \left. + \frac{a}{2b+3}\left(T_L^{2b+3} - t_w^{2b+3}\right)\right] + X\left[T_L - t_w - \frac{a}{b+1}\left(T_L^{b+1} - t_w^{b+1}\right)\right] \\
& + \alpha\left[t_w(T_O - T_L) - \frac{1}{2}\left(T_O^2 - T_L^2\right) - \frac{at_w}{b+1}\left(T_O^{b+1} - T_L^{b+1}\right) \right. \\
& \left. + \frac{a}{b+1}\left(T_O^{b+2} - T_L^{b+2}\right)\right] \\
& + \frac{\beta}{2}\left[t_w^2(T_O - T_L) - \frac{1}{3}\left(T_O^3 - T_L^3\right) - \frac{at_w^2}{b+1}\left(T_O^{b+1} - T_L^{b+1}\right) \right. \\
& \left. + \frac{a}{b+3}\left(T_O^{b+3} - T_L^{b+3}\right)\right] \\
& + \frac{a\alpha}{b+1}\left[t_w^{b+1}(T_O - T_L) - \frac{1}{b+2}\left(T_O^{b+2} - T_L^{b+2}\right) - \frac{at_w^{b+1}}{b+1}\left(T_O^{b+1} - T_L^{b+1}\right) \right. \\
& \left. + \frac{a}{2b+2}\left(T_0^{2b+2} - T_l^{2b+2}\right)\right] \\
& + \frac{a\beta}{b+2}\left[t_w^{b+2}(T_0 - T_l) - \frac{1}{b+3}\left(T_0^{b+3} - T_l^{b+3}\right) - \frac{at_w^{b+2}}{b+2}\left(T_0^{b+1} - T_l^{b+1}\right) \right. \\
& \left. + \frac{a}{2b+3}\left(T_O^{2b+3} - T_L^{2b+3}\right)\right] \\
& + \Delta d\left[\frac{1}{2}\left(T_O^2 - T_L^2\right) - T_L(T_O - T_L) - \frac{a}{b+2}\left(T_O^{b+2} - T_L^{b+2}\right) \right. \\
& \left. + \frac{T_L}{b+1}\left(T_O^{b+1} - T_L^{b+1}\right)\right] + X\left[T_O - T_L + \frac{a}{b+1}\left(T_O^{b+1} - T_L^{b+1}\right)\right]\Bigg].
\end{aligned}
$$

X. Purchasing costs per cycle is C_pQ.

XI. Total degradation cost is (C_D).

$$C_D = C_P\left[\int_0^{t_w} abt^{b-1}I_{O1}(t)dt + \int_0^{t_w} ab\,t^{b-1}I_{r1}(t)dt + \int_{t_w}^{T_L} ab\,t^{b-1}I_{r2}(t)dt + \int_{T_L}^{T_O} ab\,t^{b-1}I_{r3}(t)dt + \int_{T_O}^{T} ab\,t^{b-1}I_{r4}(t)dt\right]. \tag{15}$$

$$\begin{aligned}C_D = C_p\Bigg[&\int_0^{t_w} ab\,t^{b-1}e^{-at^{b-1}}\left[W - t\left(\alpha + \frac{\beta t}{2}\right) - at^{b+1}\left(\frac{\alpha}{b+1} + \frac{\beta t}{b+2}\right)\right]dt\\
&+ \int_0^{t_w} ab\,t^{b-1}Xe^{-at^b}dt\\
&+ \int_{t_w}^{T_L} ab\,t^{b-1}e^{-at^b}\bigg[\alpha(t_w - t) + \frac{\beta}{2}(t_w^2 - t^2)\\
&+ \frac{\alpha a}{b+1}\left(t_w^{b+1} - t^{b-1}\right) + \frac{\beta a}{b+2}\left(t_w^{b+2} - t^{b+2}\right) + X\bigg]dt\\
&+ \int_{T_L}^{T_O} ab\,t^{b-1}e^{-at^b}\bigg[X\\
&+ (\Delta d - \alpha)\left(t - T_L + \frac{a}{b+1}\left(t^{b+1} - T_L^{b+1}\right)\right)\\
&- \beta\left(\frac{1}{2}(t^2 - T_L^2) + \frac{a}{b+2}\left(t^{b+2} - T_L^{b+2}\right)\right)\bigg]dt\\
&+ \int_{T_O}^{T} ab\,t^{b-1}e^{-at^b}\bigg[(\alpha + \Delta d_1)\left((T - t) + \frac{a}{b+1}\left(T^{b+1} - t^{b+1}\right)\right)\\
&+ \beta\left(\frac{1}{2}(T^2 - t^2) + \frac{a}{b+2}\left(T^{b+2} - t^{b+2}\right)\right)\bigg]dt\Bigg].\end{aligned}$$

$$\begin{aligned}C_D = C_p\Bigg[&Wat_w^b\left(1 - \frac{at_w^b}{2}\right) - \frac{\alpha abt_w^{b+1}}{b+1}\left(1 + \frac{at_w^b}{2b+1}\right) - \frac{\beta abt_w^{b+2}}{2(b+2)}\left(1 + \frac{at_w^b}{b+1}\right)\\
&+ \alpha a^2bt_w^{2b+1}\left(\frac{1}{2b+1} + \frac{at_w^b}{(b+1)(3b+1)}\right)\\
&+ \beta a^2bt_w^{2b+2}\left(\frac{1}{4(b+1)} + \frac{at_w^b}{(b+2)(3b+2)}\right) + Xat_w^b\left(1 - \frac{at_w^b}{2}\right)\\
&+ ab\Bigg[\alpha\bigg[\frac{t_w}{b}\left(T_L^b - t_w^b\right) - \frac{at_w}{2b}\left(T_L^{2b} - t_w^{2b}\right) - \frac{1}{b+1}\left(T_L^{b+1} - t_w^{b+1}\right)\\
&+ \frac{a}{2b+1}\left(T_L^{2b+1} - t_w^{2b+1}\right)\bigg]\\
&+ \frac{\beta}{2}\bigg[\frac{t_w^2}{b}\left(T_L^b - t_w^b\right) - \frac{at_w^2}{2b}\left(T_L^{2b} - t_w^{2b}\right) - \frac{1}{b+2}\left(T_L^{b+2} - t_w^{b+2}\right)\\
&+ \frac{a}{2b+2}\left(T_L^{2b+2} - t_w^{2b+2}\right)\bigg]\Bigg]\end{aligned} \tag{16}$$

$$
\begin{aligned}
&+\frac{\alpha a}{b+1}\left[\frac{t_w^{b+1}}{b}\left(T_L^b-t_w^b\right)-\frac{at_w^{b+1}}{2b}\left(T_L^{2b}-t_w^{2b}\right)-\frac{1}{2b+1}\left(T_L^{2b+1}-t_w^{2b+1}\right)\right.\\
&\qquad\left.+\frac{a}{3b+1}\left(T_L^{3b+1}-t_w^{3b+1}\right)\right]\\
&\qquad+\frac{\beta a}{b+2}\left[\frac{t_w^{b+2}}{b}\left(T_L^b-t_w^b\right)-\frac{at_w^{b+2}}{2b}\left(T_L^{2b}-t_w^{2b}\right)-\frac{1}{2b+2}\left(T_L^{2b+2}-t_w^{2b+2}\right)\right.\\
&\qquad\left.+\frac{a}{3b+2}\left(T_L^{3b+2}-t_w^{3b+2}\right)\right]+X\left[\frac{1}{b}\left(T_L^b-t_w^b\right)-\frac{a}{2b}\left(T_L^{2b}-t_w^{2b}\right)\right]\\
&\qquad+ab\left[\alpha\left[\frac{t_w}{b}\left(T_0^b-T_L^b\right)-\frac{at_w}{2b}\left(T_0^{2b}-T_L^{2b}\right)-\frac{1}{b+1}\left(T_0^{b+1}-T_L^{b+1}\right)\right.\right.\\
&\qquad\left.+\frac{a}{2b+1}\left(T_0^{2b+1}-T_L^{2b+1}\right)\right]\\
&\qquad+\frac{\beta}{2}\left[\frac{t_w^2}{b}\left(T_0^b-T_L^b\right)-\frac{at_w^2}{2b}\left(T_0^{2b}-T_L^{2b}\right)-\frac{1}{b+2}\left(T_0^{b+2}-T_L^{b+2}\right)\right.\\
&\qquad\left.+\frac{a}{2b+2}\left(T_0^{2b+2}-T_L^{2b+2}\right)\right]\\
&\qquad+\frac{\alpha a}{b+1}\left[\frac{t_w^{b+1}}{b}\left(T_0^b-T_L^b\right)-\frac{at_w^{b+1}}{2b}\left(T_0^{2b}-T_L^{2b}\right)-\frac{1}{2b+1}\left(T_0^{2b+1}-T_L^{2b+1}\right)\right.\\
&\qquad\left.+\frac{a}{3b+1}\left(T_0^{3b+1}-T_L^{3b+1}\right)\right]\\
&\qquad+\frac{\beta a}{b+2}\left[\frac{t_w^{b+2}}{b}\left(T_0^b-T_L^b\right)-\frac{at_w^{b+2}}{2b}\left(T_0^{2b}-T_L^{2b}\right)-\frac{1}{2b+2}\left(T_0^{2b+2}-T_L^{2b+2}\right)\right.\\
&\qquad\left.+\frac{a}{3b+2}\left(T_0^{3b+2}-T_L^{3b+2}\right)\right]\\
&\qquad+\Delta d\left[\frac{1}{b+1}\left(T_0^{b+1}-T_L^{b+1}\right)\frac{a}{2b+1}\left(T_0^{2b+1}-T_L^{2b+1}\right)-\frac{T_l}{b}\left(T_0^b-T_L^b\right)\right.\\
&\qquad\left.+\frac{aT_l}{2b}\left(T_0^{2b}-T_L^{2b}\right)\right]\\
&\qquad+\Delta da\left[\frac{1}{2b+1}\left(T_0^{2b+1}-T_L^{2b+1}\right)-\frac{a}{3b+1}\left(T_0^{3b+1}-T_L^{3b+1}\right)-\frac{T_l^{b+1}}{b}\left(T_0^b-T_L^b\right)\right.\\
&\qquad\left.\left.+\frac{aT_l^{b+1}}{2b}\left(T_0^{2b}-T_L^{2b}\right)\right]+X\left[\frac{1}{b}\left(T_0^b-T_L^b\right)+\frac{a}{2b}\left(T_0^{2b}-T_L^{2b}\right)\right]\right]\\
&\qquad+ab(\Delta d_1+\alpha)\left[\frac{T}{b}\left(1-\frac{aT^b}{b+1}\right)\left(\left(T^b-T_0^b\right)-\frac{1}{2}\left(T^{2b}-T_0^{2b}\right)\right)\right.\\
&\qquad+\left(1+\frac{a}{b+1}\right)\left(\frac{T^{2b+1}-T_0^{2b+1}}{2b+1}-\frac{T^{b+1}-T_0^{b+1}}{b+1}+\frac{a\left(T^{3b+1}-T_0^{3b+1}\right)}{(b+1)(3b+1)}\right)\\
&\qquad+\beta a^2b\left(\frac{T^2\left(T^{2b}-T_0^{2b}\right)}{2b}\left(\frac{1}{2}-\frac{aT^b}{b+2}\right)-\frac{T^{2b+2}-T_0^{2b+1}}{4(b+1)}\right.\\
&\qquad\left.\left.-\frac{a\left(T^{3b+2}-T_0^{3b+2}\right)}{(b+2)(3b+2)}\right)\right]\Big].
\end{aligned}
$$

Total cost per cycle for the given model,

$$\mathrm{TC}=\mathrm{OC}+C_H+C_pQ+C_D, \tag{17}$$

where O_C, C_PQ, C_H and C_D are consider for total ordering costs, purchasing costs, holding costs and degradation costs for the given model respectively.

3 | Solution Procedure

The entire cost of the inventory system is calculated using the following steps, and the graph shows how different factors affect the system overall cost.

Step 1. Configure each input parameter.

Step 2. Analyze the pace at which demand is changing $D(t)$.

Step 3. Determine how many items are stored in RW (X).

Step 4. Analyze the stock quantity in OW and RW at every given time t, I_{01}, and $I_{r1}, I_{r2}, I_{r3}, I_{r4}$ respectively.

Step 5. Calculate t_w.

Step 6. Determine the Total Costs (TC) per cycle.

Step 7. Display the total cost curves based on several factors.

3.1 | Numerical Example

A numerical example is used to illustrate the impact of demand interruption on total costs as a result of the lock-down that was implemented by government. Using the supplied data, the graphs are solved and plotted using mathematical.

$$\text{Let } a = 0.5, b = 2, Z = 0.4, Q = 400, X = 250, W = 150, \Delta d = 130, \Delta d_1 = 70, \alpha = 230, \beta = 20, E = 3, G = 2, T_L = 0.6, T_O = T_L + Z = 1, B = 300, C = 20, T = 1.2, \text{ then TC (Total Cost)} = 10871.5. \quad \textbf{(17)}$$

4 | Sensitivity Analysis

Here we use two demand parameters like α (the starting demand rate), β (the rate at which demand rises over time) and two Weibull distribution degradation parameters like a (scale parameter), b (shape parameter). And we use another two must important factors i.e., Δd (reduced demand rate due to, lock-down imposed), Δd_1 (rise in demand rate due relax the lock-down). We consider G as holding costs per unit stocks in OW and E as holding costs per unit stocks in RW. The lock-down period $Z = (T_O - T_L)$ plays an important role in this model. Here 20% and 10% changes in parameter consider and also percentage changes in total cost are calculated in *Table 3* and graphical sensitivity analysis shown in *Fig. 2*.

I. We can see that any modification to these factors has a substantial impact on the result. When the lock-down period Z increases, the degradation cost as well as the holding cost increases So the total cost increases.

II. The parameter α and β are related to demand function, so when α or β increases the demand is increasing. Therefore, effect of degradation decreases and the holding time of stocks at both warehouse decreases which implies a decrease in total degradation cost and holding cost. So total cost decreases.

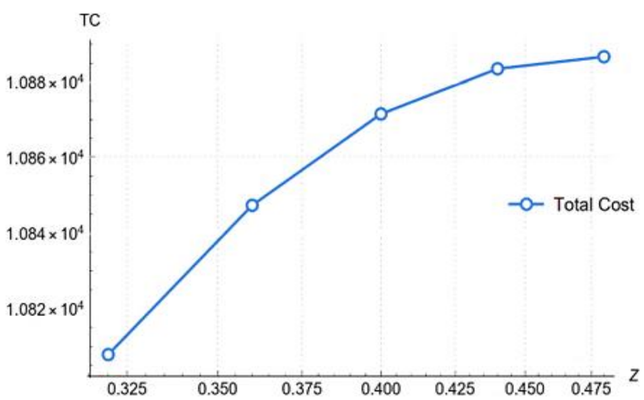
III. When the holding costs per unit item of OW (G) and RW (E) rises, the total holding costs also rises while other costs remain constant, which results in an increase in the total costs.

IV. Δd is the amount of demand decrease due to lock-down imposed by government. So, when Δd rises the demand rate fall so stocks are kept at warehouse for a longer time and effect of degradation increases and due to this, the holding cost and degradation cost was increases. Therefore, total cost of the model increases.

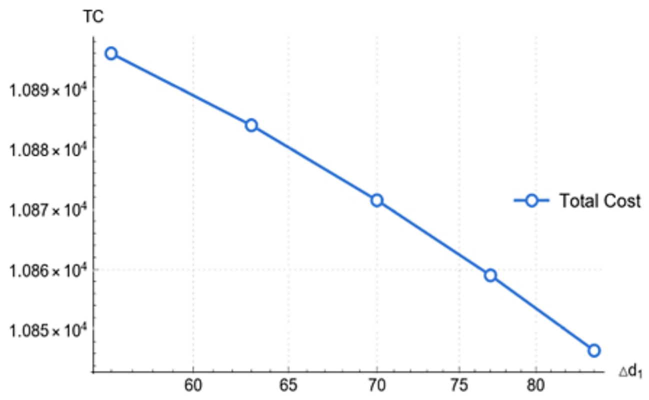
V. Lock-down was relaxed by government so Δd_1 amount of demand increases because panic buying behaviour of the customer. So, when Δd_1 increases, the holding period of stocks at warehouse decreases and effect of degradation decreases. So, the total holding costs and degradation cost decreases. Therefore, total costs of the model decreases.

Table 3. Sensitivity analysis for the research.

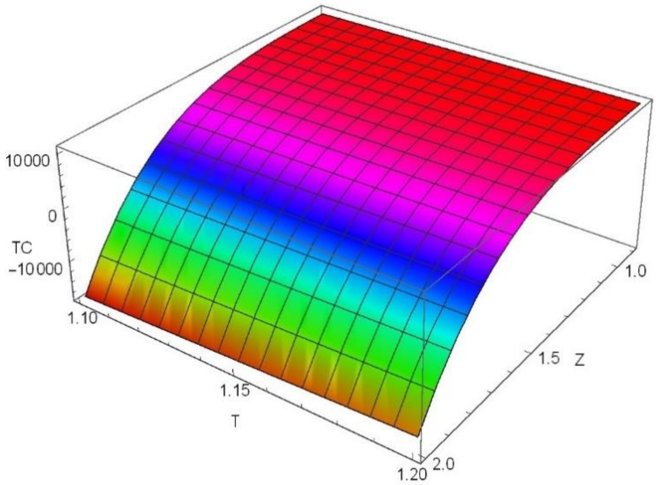
Parameter	Value	Total Cost	Total Cost Variation (%)
a	0.6	11268.6	3.65
	0.55	11068	1.81
	0.45	10678.2	-1.78
	0.4	10488.1	-3.53
b	2.4	10922.4	0.47
	2.2	10896.5	0.23
	1.8	10847.8	-0.22
	1.6	10825.6	-0.46
Δd	156	10900	0.26
	143	10885.8	0.13
	117	10857.3	-0.13
	104	10843	-0.26
Δd_1	84	10846.5	-0.23
	77	10859	-0.11
	63	10884	0.11
	56	10896	0.23
α	276	10805.2	-0.61
	253	10838.4	-0.3
	207	10904.7	0.31
	184	10925.8	0.5
β	24	10799.8	-0.66
	22	10865.7	-0.05
	18	10877.3	0.05
	16	10883.2	0.11
Z	0.48	10886.7	0.14
	0.44	10883.5	0.11
	0.36	10847.6	-0.22
	0.32	10807.9	-0.59
G	2.4	10887.6	0.15
	2.2	10879.6	0.07
	1.8	10863.4	-0.07
	1.6	10855.4	-0.15
E	3.6	10967.1	0.88
	3.3	10919.3	0.44
	2.7	10823.7	-0.44
	2.4	10775.9	-0.88



a. Total cost (TC) varies with respect to Δd.



b. Total cost (TC) varies with respect to Δd_1.



c. Total cost (TC) varies with respect to Z and time T.

Fig 2. Total cost (TC) change with respect to Δd, Δd_1 and Z with respect to T.

5 | Conclusion

In the majority of countries, the Covid-19 outbreak has dramatically changed the economic landscape. During Covid-19 epidemic, many victims are owners of medium- and small-sized businesses. A country's economy will expand in the business sector, which is the pillar of the economy. This article investigates the effect Covid-19 epidemic and suggests an inventory approach. This mathematical research was investigating, (a) the impact of demand interruption, i.e., the decline and increasing demands due to Covid-19 lock-down, (b) the effect of time frame of lock-down on the total costs, and (c) importance of two warehouse inventory system. We solve a mathematical scenario to show how the lock-down duration influences total expenditures. This research examines the importance of FIFO policy to measure the effect of degradation. Due to discounts provided for big orders as well as other factors, the merchant makes more orders than the warehouse can store. The merchant is therefore needed to keep the excess supplies in any RW in this model. This research demonstrates that increasing OW capacity may lower total holding costs since RW has higher holding costs than OW, hence lowering the system's overall cost. It has a number of limitations, for instance, it ignores inflation and partial backlogs and employs a fixed rise and drop in convenience demand because genuine demand disruption is difficult to assess.

Author Contributions

Problem finding, article writing and sensitivity analysis: D. Das; Article reviewing, designing and editing: G. C. Samanta

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Data Availability

No external data use for this research work.

Conflicts of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article. All authors contributed equally to this research work.

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