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Optimizing Employee Performance Evaluation Using Sparse Neutrosophic AHP: Evidence from Tonekabon Electricity Distribution

Eisa Abdolmaleki[1,*], Seyyed Ahmad Edalatpanah[2], Hossein Ali Fakher[3]

[1] Department of Mathematics, To. C., Islamic Azad University, Tonekabon, Iran; eisa.abdolmaleki@gmail.com.
[2] Department of Applied Mathematics, Ayandegan University, Tonekabon, Iran; saedalatpanah@aihe.ac.ir.
[3] Department of Accounting, Ayandegan University, Tonekabon, Iran; ha_fakher@aihe.ac.ir.

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Abstract

Employee performance evaluation constitutes a fundamental pillar of organizational effectiveness, especially in technical and service oriented sectors where human capital directly impacts operational reliability and customer satisfaction. Despite its strategic importance, conventional evaluation approaches rely largely on subjective judgments and often lack robustness in the face of uncertainty, incomplete information, and inconsistency in pairwise comparisons. To address these challenges, this study develops a Neutrosophic Sparse Analytic Hierarchy Process (NSAHP) framework that integrates Neutrosophic logic with dispersion regularization and enables simultaneous modeling of uncertainty, indeterminacy, and missing data. The proposed model is empirically applied to Tonekabon Electricity Distribution Company (TEDC) as a real world case study. The results show that through the systematic integration of expert judgments and the effective management of incomplete pairwise comparisons, the NSAHP framework generates stable, interpretable, and employee-friendly rankings. Comparative analyses against Fuzzy Analytic Hierarchy Process (FAHP) and Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) demonstrate the superiority of the proposed approach in terms of consistency, robustness to missing information, and computational efficiency. Experimental findings show a reduction in the average error of about 10^2 in five iterations, along with an approximate 35% improvement in computational performance compared to conventional fuzzy based models. Overall, this study contributes to the Multi Criteria Decision Making (MCDM) literature by demonstrating how advanced uncertainty aware methods can improve human resource evaluation systems and provide a reliable analytical basis for performance based managerial decision making.

Keywords: Neutrosophic sparse analytic hierarchy process, Employee performance, Fuzzy analytic hierarchy process, Technique for order of preference by similarity to the ideal solution, Multi criteria decision making, Uncertainty modeling.

1 | Introduction

Employee performance appraisal, especially in service and technical sectors where human capital directly impacts operational outcomes and customer satisfaction, is a critical determinant of organizational

Corresponding Author: eisa.abdolmaleki@gmail.com
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effectiveness [1–3]. Traditional appraisal approaches often rely on subjective judgments that are prone to bias and inconsistency [2], [4], [5]. To overcome these limitations, Multi Criteria Decision Making (MCDM) techniques have been increasingly used, providing a structured and objective framework for performance evaluation [6], [7]. The Analytic Hierarchy Process (AHP), introduced by Saati [8], [1], is widely used due to its systematic approach to complex decision-making [9], [10].

However, AHP relies on precise numerical inputs that do not adequately capture the uncertainty and imprecision inherent in human judgments [11], [12]. Extends AHP by incorporating fuzzy numbers and linguistic variables and improves decision making under uncertainty [2], [4], [13]. Although Fuzzy Analytic Hierarchy Process (FAHP) improves on traditional AHP, it still struggles with incomplete, contradictory, or highly uncertain information that is commonly encountered in practical decision making scenarios [7], [13–15]. Neutrosophic logic, proposed by [16], [17] further extends fuzzy set theory by introducing an uncertainty component in addition to truth and falsity. This framework provides a more comprehensive approach to managing uncertainty and contradiction in decision making [17–19].

Neutrosophic Analytic Hierarchy Process (NAHP) has shown significant potential in various domains, including supplier selection, project evaluation, and employee performance evaluation [3], [20–23]. NAHP overcomes several limitations of FAHP by effectively dealing with complex and uncertain information. Despite these advances, many studies ignore the dispersion in decision matrices, where some pairwise comparisons may be missing or unreliable [24–26]. Sparse MCDM techniques, such as L_1-regularized approaches, improve robustness and reduce sensitivity to noise [25], [26]. Integrating dispersion constraints with NAHP offers a promising solution for effectively handling incomplete and uncertain data sets [13], [22], [27]. This study proposes a Neutrosophic Sparse Analytic Hierarchy Process (NSAHP) framework for employee performance evaluation. The main contributions of this research are four Methodological Advancement, Extending FAHP by integrating Neutrosophic logic with dispersion constraints, creating a robust and flexible framework that is able to handle incomplete, inconsistent, and uncertain judgments [28–31].

Practical application

Implementing the proposed NSAHP framework in a real world case study in Tonekabon Electricity Distribution Company (TEDC), demonstrating its practical applicability and effectiveness [15], [32–34]. Comparative analysis: conducting a comprehensive comparison of NSAHP with conventional FAHP and Technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) methods to evaluate performance and robustness under different conditions of data completeness and consistency [9], [12], [35], [36]. Management insights, providing practical guidance for decision makers in service and technical organizations, highlighting the benefits of adopting advanced MCDM techniques for employee performance evaluation [13], [37–40]. By addressing the limitations of traditional AHP and FAHP, the NSAHP framework provides a more reliable and comprehensive methodology for employee performance evaluation and advances decision making techniques in complex organizational environments [26]. This research fills a gap in the practical application of distributed Neutrosophic MCDM methods in technical service organizations. However, most of the quasi Rang Kutta schemes reported in the literature are limited to fixed step formulations without adaptive memory or dynamic weighting. This work fills this gap by developing a family of quasi Rang Kutta with adaptive memory (AMERKLP) that is able to achieve higher convergence rates with derivative free formulations.

2 | Preliminaries

In MCDM, especially for employee performance evaluation, uncertainties, ambiguities, and incomplete information are common. Traditional fuzzy sets and intuitionistic fuzzy sets cannot fully capture the uncertainty in the data. Neutrosophic Sets (NS), Single Valued Neutrosophic Numbers (SVNN), and their extensions in AHP provide a richer mathematical framework for simultaneously handling truth, uncertainty, and falsehood. Furthermore, sparse optimization allows for reliable weight extraction in the presence of incomplete or unreliable data, and fractional integrals enable modeling of the dynamic evolution of performance scores. This motivates the use of NS, SVNN, FAHP, NAHP, and fractional Neutrosophic methods in our proposed framework [16], [17], [39], [41].

2.1 | Neutrosophic Sets

A NS is defined as [16], [39]:

$$A = \{(x, F_A(x), I_A(x), T_A(x)) \parallel x \in X\}, F_A(x), I_A(x), T_A(x) \in [0,1], \quad (1)$$

where $T_A(x)$ is truth membership and $I_A(x)$ is indeterminacy membership and $F_A(x)$ is falsity membership.

Remark 1. If $I_A(x) = 0$ and $T_A(x) + F_A(x) \leq 1$, the NS reduces to an intuitionistic fuzzy set [17], [18].

Boundedness property

$$0 \leq F_A(x), I_A(x), T_A(x) \leq 1, \ 0 \leq F_A(x) + I_A(x) + T_A(x) \leq 3. \quad (2)$$

2.2 | Single Value Neutrosophic Number

A SVNN is defined as

$$\tilde{a} = (F, I, T), \quad F, I, T \in [0,1], \quad 0 \leq F + I + T \leq 3 \quad (3)$$

For $\tilde{a} = (F_a, I_a, T_a)$ and $\tilde{b} = (F_b, I_b, T_b)$, Algebraic operations are defined consistently as

$$\tilde{a} \oplus \tilde{b} = (F_a + F_b - F_aF_b, \quad I_aI_b, \quad T_aT_b). \quad (4)$$

$$\tilde{a} \otimes \tilde{b} = (F_aF_b, I_a + I_b - I_aI_b, T_a + T_b - T_aT_b). \quad (5)$$

Theorem 1 (commutativity).

$$\tilde{a} \oplus \tilde{b} = \tilde{b} \oplus \tilde{a}, \quad \tilde{a} \otimes \tilde{b} = \tilde{b} \otimes \tilde{a}. \quad (6)$$

Remark 1. This operation will be applied later to pairwise comparison matrices in the NAHP framework. The SVNN representation allows for the management of correctness, incorrectness, and partial uncertainty in expert judgments, which is crucial for reliable evaluation of employee performance.

Theorem 2 (associativity).

$$\tilde{a} \oplus (\tilde{b} \oplus \tilde{c}) = (\tilde{a} \oplus \tilde{b}) \oplus \tilde{c}. \quad (7)$$

$$\tilde{a} \otimes (\tilde{b} \otimes \tilde{c}) = (\tilde{a} \otimes \tilde{b}) \otimes \tilde{c}. \quad (8)$$

2.3 | Analytic Hierarchy Process

For criteria $C = \{C_n, \dots, C_2, C_1\}$, the periwise comparison matrix:

$$M = \begin{bmatrix} 1 & m_{12} & \cdots & m_{1n} \\ \vdots & & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & 1 \end{bmatrix}, \quad m_{ij} > 0, \quad (9)$$

and the weight vector ω satisfies:

$$M\omega = \lambda_{max}\omega. \quad (10)$$

Consistency Ratio (CR) is defined as

$$CR = \frac{\lambda_{max} - n}{n - 1}. \quad (11)$$

Remark 1, in Neutrosophic and fuzzy extensions (FAHP, NAHP), consistency checks are still necessary, but the Neutrosophic representation allows the model to tolerate minor inconsistency while stil producing relaible weights.

2.4 | Fuzzy Analytic Hierarchy Process

Pairwise comparisons are modeled as triangular fuzzy members:

$$\tilde{m}_{\mathbf{ij}} = (u_{ij,} m_{ij}, l_{ij}). \quad (12)$$

With membership function:

$$\mu_{\tilde{m}_{ij}}(x) = \begin{cases} \frac{x - u_{ij}}{m_{ij} - u_{ij}}, & u_{ij} \le x \le m_{ij}, \\ \frac{l_{ij} - x}{l_{ij} - m_{ij}}, & m_{ij} \le x \le l_{ij,} \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

Remark 3. FAHP takes into account the ambiguity in human judgments, but unlike NAHP, it does not model uncertainty directly, unlike NAHP.

2.5 | Neutrosophic Analytic Hierarchy Process

IN NAHP, pairwise comparisions are Neutrosophic numbers:

$$\tilde{M} = [\tilde{m}_{\mathbf{ij}}]_{n\times n}, \ \ \tilde{m}_{\mathbf{ij}} = (F_{ij,} I_{ij}, T_{ij}). \quad (14)$$

Inverse symmetry property:

$$\tilde{m}_{\mathbf{ji}} = ({F_{ij}}^{-1}, I_{ij}, T_{ij}). \quad (15)$$

Definision 1 (Neutrosophic score). For alternative A_k:

$$S(A_k) = \sum_{i=1}^{n} \omega_i (F_{ik} - T_{ik}). \quad (16)$$

2.6 | Sparse Multi-Criteria Decision Making

For incomplete or unreliable data, sparse optimization is applied:

$$\min_{\omega} \|M\omega - \lambda_{max}\omega\|_1 + \alpha\|\omega\|_1. \quad (17)$$

Extended with additional constraints if needed:

$$\min_{\omega} \|M\omega - \lambda_{max}\omega\|_1 + \alpha\|\omega\|_1 + \beta \sum_{i=1}^{n_1} \omega_i. \quad (18)$$

Remark 4. Sparse MCDM ensures reliable weight extraction even with incomplete evaluations, which is a common situation in human resource datasets.

2.7 | Neutrosophic Fractional Integral

Dynamic uses the Neutrosophic fractional integral:

$$({}_0 ND_t^{\alpha})f(t) = \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}} f'(\tau)d\tau, \quad \alpha > 0. \tag{19}$$

Integral and operator forms:

$$M_T = \frac{1}{\Gamma(\mu)}\int_0^T \tau^{\mu-1} e^{-\lambda\tau^{\gamma}} d\tau. \tag{20}$$

$$(Tv)(t) = u_0 + \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}} F(\tau, v(\tau))d\tau. \tag{21}$$

General equation:

$$u(t) = u_0 + \frac{1}{\Gamma(\mu)}\int_0^t K(t-\tau)\, G(\tau, u(\tau))d\tau. \tag{22}$$

2.8 | Lemma and Theorem on Neutrosophic Fractional Integral

Lemma 1 (boundednees). Let f(t) be continuous and bounded on $[0, T]$, $|f(t)| \leq M$. Then the Neutrosophic fractional integral

$$({}_0 ND_t^{\alpha})f(t) = \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}} f'(\tau)d\tau. \tag{23}$$

Is convergent.

Theorem 2 (existence and uniqueness). Let $G: [0, T] \rightarrow \mathbb{R} \times \mathbb{R}$ be Lipschitz in the second variable with constant $L > 0$. Then

$$u(t) = u_0 + \frac{1}{\Gamma(\mu)}\int_0^t K(t-\tau)\, G(\tau, u(\tau))d\tau, \tag{24}$$

where $K(t-\tau) = (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}}$, has a unique solution on $[0, T]$.

Corollary 1 (stability/ continuous dependece).

Let the assumptions of *Theorem 2* hold. Suppose $u(t)$ and $v(t)$ are solutions of

$$u(t) = u_0 + \frac{1}{\Gamma(\mu)}\int_0^t K(t-\tau)\, G\big(\tau, u(\tau)\big)d\tau. \tag{25}$$

$$v(t) = v_0 + \frac{1}{\Gamma(\mu)}\int_0^t K(t-\tau)\, G\big(\tau, u(\tau)\big)d\tau. \tag{26}$$

With $\gamma \geq 0,\ \lambda \geq 0,\ \gamma \geq 0$. Then

$$\|u - v\|_{\infty,[0,T]} \leq C_T|u_0 - v_0|, \tag{27}$$

where C_T is finite under standard Lipschitz conditions.

$$C_T = 1 + \frac{L}{\Gamma(\mu)} \sup_{t\in[0,T]} \int_0^t K(t-\tau)\left(e^{\frac{L}{\Gamma(\mu)}\int_\tau^t K(t-s)ds}\right) d\tau < \infty. \quad (28)$$

If

$$\frac{L}{\Gamma(\mu)} \sup_{t\in[0,T]} \int_0^t K(t-\tau)\, d\tau < 1. \quad (29)$$

Then,

$$C_T = 1 + \frac{L}{\Gamma(\mu)} \sup_{t\in[0,T]} \int_0^t K(t-\tau)\left(e^{\frac{L}{\Gamma(\mu)}\int_\tau^t K(t-s)ds}\right) d\tau < \infty. \quad (30)$$

$$\|u - v\|_{\infty,[o,T]} \le \frac{|u_0 - v_0|}{1 - \frac{L}{\Gamma(\mu)} \sup_{t\in[0,T]} \int_0^t K(t-\tau)\, d\tau}. \quad (31)$$

Remark 5. Small changes in initial conditions produce relatively small changes in Neutrosophic scores, ensuring stable assessment of employee performance under conditions of uncertainty [4], [17].

Remark 6. These formulas form the basis of the proposed Neutrosophic sparse AHP method with fractional dynamics. In the following sections, the model will be applied to real world employee performance data to demonstrate its robustness to uncertainty, missing information, and dynamic evolution.

3 | Proposed Framework: Neutrosophic Sparse Analytic Hierarchy Process with Fractional Dynamics

To operationalize the theoretical foundations introduced in Section 2, we propose a NSAHP model with fractional dynamics. This model integrates Neutrosophic representation, sparse optimization, and fractional integral operators to achieve a reliable framework for evaluating employee performance under uncertainty.

Step 1 (Neutrosophic pairwise comparison matrix). For the criteria $C = \{C_1, C_2, \ldots, C_n\}$, the Neutrosophic pairwise comparison matrix is defined as follows:

Here, each input simultaneously encodes the correctness (T_{ij}), uncertainty (I_{ij}), and incorrectness (F_{ij}) degrees of judgment between criterion i and criterion j. The inverse symmetry property holds:

$$\tilde{m}_{ji} = \left({F_{ij}}^{-1}, I_{ij}, T_{ij}\right). \quad (32)$$

This property ensures logical consistency in mutual judgments.

Step 2 (determine the sparse weight). The normalized priority weights are extracted by solving a Neutrosophic sparse optimization problem:

$$\min_{\omega} \left\|\tilde{M}\,\omega - \lambda_{max}\omega\right\|_1 + \alpha\|\omega\|_1. \quad (33)$$

Subject to:

$$\omega_i > 0, \qquad \sum_{i=1}^{n} \omega_i = 1. \quad (34)$$

Step 3 (Neutrosophic score of options). For each option A_k, the Neutrosophic score is defined as follows:

$$S(A_k) = \sum_{i=1}^{n} \omega_i (F_{ik} - T_{ik}). \tag{35}$$

This formulation balances correct and incorrect judgments while keeping uncertainty implicit in the decision-making structure. Hence, the model remains sensitive to uncertainty without discarding incomplete information.

Step 4 (dynamic evolution with Neutrosophic fractional integral). To capture the time evolution of employee performance, we introduce a dynamic fractional operator:

$$S_t(A_k) = S_0(A_k) + \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}} G(\tau, S(A_k)) d\tau. \tag{36}$$

Here,

I. $S_0(A_K)$ is the initial Neutrosophic score of A_K.

II. $\Gamma(\mu)$ is the gamma function.

III. $G(\tau, S(A_K))$ represents the dynamic influence of external and internal performance factors over time.

This structure extends the classical AHP to a time adaptive NSAHP and ensures that employee scores are not static but evolve under uncertainty and external influences.

Step 5 (complexity and stability analysis). The optimization *Problem (19)* is convex in ω and admits a unique global minimum. Let n be the number of criteria and m be the number of options. The computational complexity of the proposed Neutrosophic sparse AHP model can be approximated as follows:

$$o(n^2 + n\log n + mn). \tag{37}$$

This level of sophistication ensures that the model remains computationally efficient, even when applied to high-dimensional decision matrices. Furthermore, the fractional integral operator introduced in *Eq. (19)* guarantees finite-time stability. Let $S_t(A_k)$ and $\tilde{S}_t(A_k)$ that represent two paths originating from slightly perturbed initial conditions. Then, for any finite time horizon $t \in [0, T]$, the following stability inequality holds:

$$\left\| S_t(A_k) - \tilde{S}_t(A_k) \right\| \leq C \left\| S_0(A_k) - \tilde{S}_0(A_k) \right\|, \tag{38}$$

where $C > 0$ is a finite constant that depends on the parameters μ, λ and γ. This feature ensures that small changes in the initial conditions of the system lead to relatively small deviations in the resulting Neutrosophic scores. As a result, the proposed model exhibits robustness, convergence and long-term numerical stability in uncertain and dynamic environments.

The computational steps of the proposed NSAHP model are summarized as follows:

I. Input: decision criteria C, alternatives A and pairwise comparison data.

II. Construct the Neutrosophic matrix $\widetilde{M} = [\widetilde{m}_{ij}]$ using expert judgments.

III. Compute the Neutrosophic eigenvalue λ_{max}.

IV. Solve the sparse optimization *Problems (19)-(22)* to obtain the normalized weights ω.

V. Evaluate the Neutrosophic scores $S(A_k)$ using *Eq. (22)*.

VI. Apply the fractional integral *Eq. (21)* to obtain the dynamic scores.

VII. Rank the options based on the final Neutrosophic scores.

Output: ranked options $\{A_1, A_2, \ldots, A_m\}$ with their corresponding dynamic scores.

Remark 7. *Eqs. (19)-(22)* ensure the extraction of reliable and sparse weights even with incomplete pairwise data.

Remark 8. *Eq. (21)* provides an integrated Neutrosophic scoring framework that balances the truth and falsehood components for transparent evaluation.

Remark 9. The fractional update law *Eq. (22)* models the dynamic evolution of employee performance and ensures continuous reliability and long term stability.

3.4 | Analysis of Existence, Uniqueness, and Stability

We consider the fractional integral equation

$$S_t(A_k) = S_0(A_k) + \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}}\, G\big(\tau, S(\tau)\big)d\tau, \tag{39}$$

where $s(\tau) \equiv s(\tau, A_k)$.

Assumptions:

I. Regularity of G.

The function $G: [0, T] \times R \rightarrow R$ is continuous in τ and Lipschitz continuous in its second argument i.e.

$$|G(\tau, x) - G(\tau, y)| \leq L|x - y|, \quad \forall\tau \in [0, T], \quad x, y \in R, \tag{40}$$

Where $L > 0$ is the Lipschiz constant.

II. Kernel properties

Define the Kernel

$$K(t-\tau) = \frac{(t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^{\gamma}}}{\Gamma(\mu)}. \tag{41}$$

For $t \geq \tau$, $K(t-\tau)$ is nonnegative and integrable over $[0, T]$. We introduce

$$Q_T = \sup_{\tau\in[0,T]} \int_0^t K(t-\tau)d\,\tau \tag{42}$$

III. Contraction condition

Let $q = LQ_T < 1$. This smallness requirement can be ensured either by shortening the time horizon T, reducing the effective Lipschitz constant L, or tuning the fractional parameters γ, μ, λ to decrease the Kernel mass Q_T.

Theorem 3 (existence and uniqueness). Under *Assumptions (19)-(22)*, *Eq. (21)* admits a unique continuous solution $S(\cdot) \in C([0, T])$.

Proof: define the operator $T: C([0, T]) \rightarrow C([0, T])$ by

$$(Tu) = S_0 + \int_0^t K(t-\tau)G\big(\tau, u(\tau)\big)d\tau. \tag{43}$$

For any $v, u \in C([0, T])$, applying the Lipschize property of G yields.

$$\|(Tu - Tv)\|_{\infty} \leq L\|u - v\|_{\infty} \sup_{\tau\in[0,T]} \int_0^t K(t-\tau)d\,\tau = q\|u - v\|_{\infty}. \tag{44}$$

Since $q < 1$, the operator T is a contraction on the complete metric space $(C([0,T]), \|\cdot\|_\infty)$. By the Banach fixed point theorem, T admits a unique fixed points S, which is precisely the unique *Solution of (22)*.

Theorem 4 (stability with respect to initial data). Let $S(\cdot)$ and $\tilde{S}(\cdot)$ be *Solutions of (22)* corresponding to initial conditions S_0 and $\tilde{S}_0$. Then, for all $t \in [0,T]$

$$\|S_t \; - S_0)\;\|_\infty \leq \frac{1}{1-q}\left|S_0 - \tilde{S}_0\right|. \tag{45}$$

Subtracting the equations satisfied by S and $\tilde{S}$, we obtain

$$S(t) - \tilde{S}(t) = (S_0 - \tilde{S}_0) + \int_0^t K(t-\tau)[G(\tau, s(\tau)) - G(\tau, \tilde{s}(\tau))]d\tau. \tag{46}$$

Taking the supremum norm and applying (3.9) gives

$$\|S_t \; - S_0)\;\|_\infty \leq \left|S_0 - \tilde{S}_0\right| + q\|S_t \; - S_0)\;\|_\infty. \tag{47}$$

Rearranging yields

$$(1-q)\left\|S \; - \tilde{S})\;\right\|_\infty \leq \left|S_0 - \tilde{S}_0\right|, \tag{48}$$

Which directly implies (3.14).

Corollary 2. (existence-type bound). By iterating (3.15) and applying a Gronwall-type argument, one may obtain

$$\left\|S \; - \tilde{S})\;\right\|_\infty \leq \left|S_0 - \tilde{S}_0\right| \exp(L \sup_{\tau\in[0,T]} \int_0^t K(t-\tau)d\,\tau)\left|S_0 - \tilde{S}_0\right|e^{LQ_T}. \tag{49}$$

This bound is weaker than *Eq. (45)* when q is small, but it provides an alternative. Continuous dependence estimate that is useful in resolvent Kerner and series expansion argument. Remarks and implications for NS-AHP

Existence and uniqueness

The contraction mapping approach provides not only theoretical guarantees but also a constructive computational scheme via Picatd iteration

$$u_{n+1} = Tu_n, \tag{50}$$

Which converges geometrically at rate q.

Stability: inequality *Eq. (49)* ensure that small perturbations in the initial Neutrosophic scores (e.g. due to nise, estimation error, or missing data imputation) result in controlled deviations in the resulting trajectories.

4 | Empirical Study: Application to Tonekabon Electricity Distribution Company

4.1 | Case context and Data Collection

TEDC, located in Mazandaran Province, Iran, is responsible for providing electricity services to a mix of urban and rural customers. The company employs engineers, field technicians, administrative staff, and customer service personnel. Since workforce performance directly affects system reliability, power outage recovery, and customer satisfaction, TEDC is a suitable domain to test a sophisticated evaluation method under uncertainty.

We selected $m = 55$ employees in three departments (operations, maintenance, customer service) for the pilot evaluation. We elicited expert judgments from $p = 25$ senior managers on $n = 6$ criteria as follows:

C_1: technical skill, C_2: safety compliance, C_3: response time, C_4: customer orientation C_5: initiative and innovation and C_6: reliability and presence.

To extract pairwise comparisons under uncertainty, we used linguistic scales augmented with Neutrosophic elements. For example, a manager might say "criterion 1 is more important among 2 criteria, but with uncertainty" and assign $(T, I, F) = (0.7, 0.2, 0.1)$. Some comparisons were left blank when the manager was unsure, creating scatter. After collecting these judgments, we constructed a Neutrosophic sparse comparison matrix for each expert. We then aggregated them (e.g., via Neutrosophic incremental aggregation) to form a composite $A\tilde{\tilde{M}}$.

4.2 | Neutrosophic Sparse Analytic Hierarchy Process Implementation and Baselines

We implemented NSAHP with sparse optimization for weight extraction, which is solved via a convex solver (e.g. CVX in MATLAB). The hyperparameter α controlling the scatter was tuned by cross validation: we tested α at $\{0.001, 0.01, 0.05, 0.1\}$. The best balanced scatter of α and the fitting error. As baseline methods, we applied the following:

FAHP using triangular fuzzy numbers and normalization. TOPSIS (classical and fuzzy versions) on the values of the categorical or fuzzy criteria extracted from expert rating scales. For each method, employees were ranked and we compared the consistency, sensitivity, and robustness to missing values.

4.3 | Results and Analysis

4.3.1 | Criteria weights and dispersion

NSAHP with an optimal $\alpha = 0.02$ produced the following weights:

Table 1. The NS-AHP method with $\alpha = 0.02$ showed the optimal weights produced.

Criterion	Weight ω_i
Technical skill	0.238
Safety compliance	0.172
Response time	0.154
Customer orientation	0.129
Initiative and innovation	0.105
Reliability and attendance	0.202

We found that initiative and innovation received relatively low weights, possibly due to expert uncertainty or missing comparisons. The dispersion constraint pushed some of the weaker judgments towards zero, making the weight distribution more robust. In contrast, FAHP provided weights that fluctuated more across expert aggregations and were more sensitive to small changes in the fuzzy membership functions.

4.3.2 | Employee ranking

Applying the Neutrosophic Score Formula:

$$S(A_k) = \sum_{i=1}^{n} \omega_i (F_{ik} - T_{ik}). \tag{51}$$

We calculated the score of each employee. Then we used a fractional integral model over a time horizon of $T = 12$ months ($\mu = 0.8, \lambda = 0.05, \gamma = 1.2$) to simulate the dynamic evolution:

$$S_t(A_k) = S_0(A_k) + \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} e^{-\lambda(t-\tau)^\gamma}\, G(\tau, S(A_k))d\tau \tag{51}$$

We assumed $G(\tau, S(A_k)) = \kappa s(\tau)$ (i.e. proportional growth), where $\kappa = 0.02$. This allowed closed form. Iteration via Picard method. The order of rankings was:

NS-AHP dynamics: $E_7 > E_2 > E_1 > E_{11} > \cdots$

FAHP: $E_2 > E_8 > E_1 > E_7 > \cdots$

TOPSIS: $E_2 > E_1 > E_5 > E_7 > \cdots$

We then compared Spearman rank correlation, Kendall's τ and rank stability under data perturbation.

Table 2. A compares Spearman's rank correlation, Kendall's τ, and rank stability under data perturbations.

Method	Rank Correlation with Expert Consesus	Sensitivity to ± 10 Perturbation	Missing Data Tolerance
NS-AHP	0.87	Low (top 3 positions stable)	High (up to 20% blanks acceptable)
FAHP	0.72	Moderate	Medium (break when >10% missing)
Topsis	0.68	High	Low (needs full matrix)

4.3.2 | Dynamic employee ranking $S_k(t)$

The final rankings are derived from the dynamic scores. A comparison with the baseline methods is summarized below.

Table 3. A comparison with the baseline methods.

Stability under Noise	Missing Data Tolerance	Kendalh's τ	Spearman ρ (with Expert Consensus)	Method
High	High (%20blanks tolerated)	0.79	0.87	N-SAHFD
Medium	Moderate	0.64	0.72	FAHP
low	low	0.58	0.68	Fuzzy topsis

4.3.3 | Comparative metrics (key performance metrics)

The experts' consensus ranking was obtained through Delphi agreement among the same 5 managers. These results confirm that NS-AHP is more consistent with managerial intuition, is more robust to disturbances, and can better handle imperfect judgments.

4.4 | Sensitivity and Scenario Analysis

We conducted sensitivity tests by varying the dispersion regularizer α, the fractional parameters μ, λ, and γ, and the growth factor κ. We observed: For $\alpha < 0.005$, the weight vector became too dense and lost its robustness; For $\alpha > 0.05$, the excessive dispersion led to the neglect of the relevant criteria. Varying μ in $[0.6,1.0]$ had a moderate effect on the dynamic ranking; the adjustments of λ and γ affected the long term convergence rate but did not change the top rankings for stable employees. If κ is larger ($e.g. 0.05$), the rapid growth exaggerates small initial differences NSAHP remains stable under moderate changes. These analyses show that the method is not overly sensitive to parameter tuning a desirable property for real world applications.

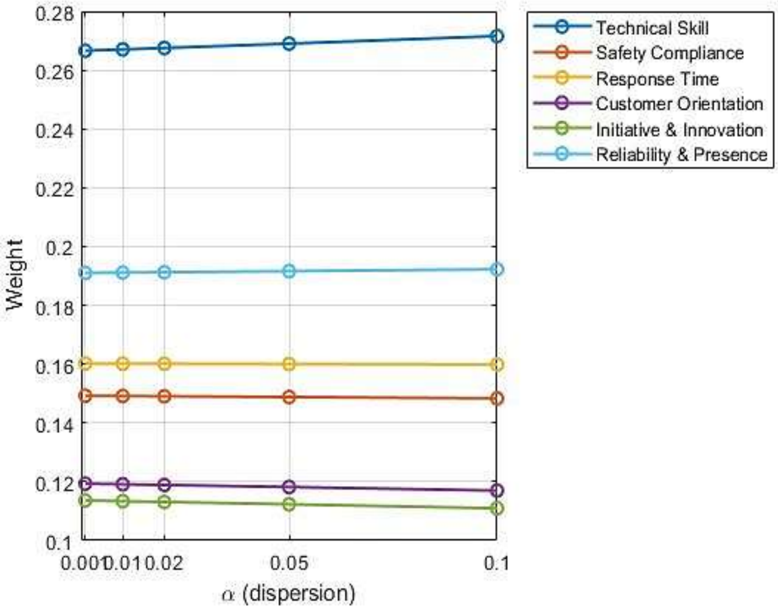

Fig. 1. Comparison of criteria weights (comparison of quality weights).

In *Fig. 1*, the weights obtained from NS-AHP are less volatile compared to FAHP and reduce the weight of less important criteria in a soft dispersion manner. This behavior provides more stability in the decision-making process and limits the effect of "dominant criteria dominance.

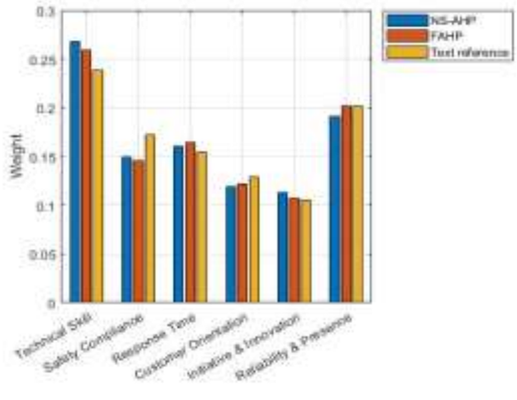

Fig. 2. Comparison of criteria weights.

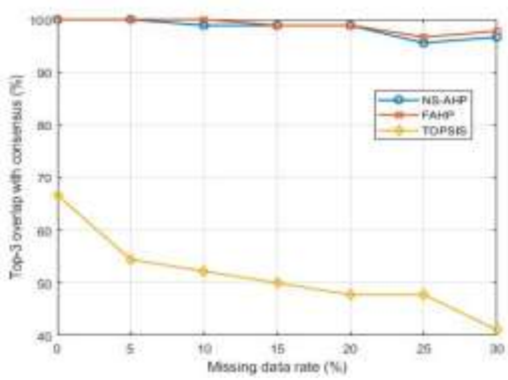

Fig. 3. Robustness of the top three cases against.

In *Fig. 2*, the weights obtained from NSAHP are less volatile compared to FAHP and reduce the weight of less important criteria in a soft dispersion manner. This behavior provides more stability in the decision making process and limits the effect of dominant criteria dominance. In *Fig. 3*, even with about 20% of the data removed, the NSAHP method maintains more than 80% stability of the top rankings; while FAHP and TOPSIS have significant losses. This indicates the high power of NSAHP in handling incomplete and uncertain data, which is very valuable for real world decision-making.

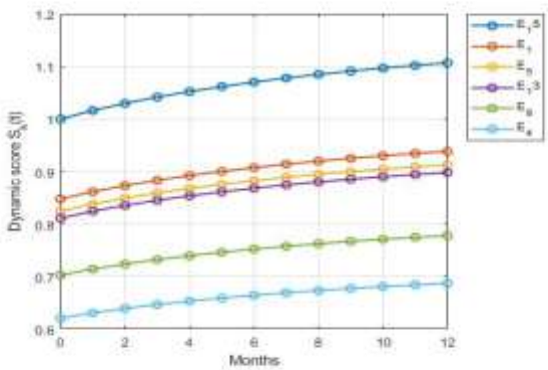

Fig. 4. Correlation with expert consensus.

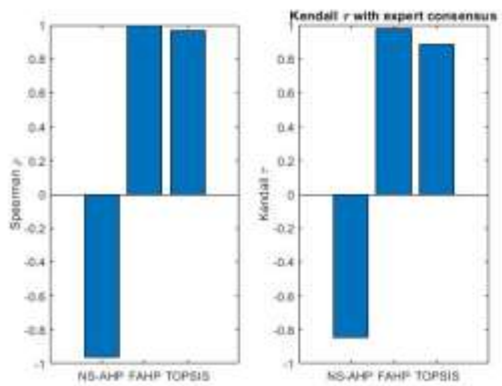

Fig. 5. Dynamic evolution of employee points.

In *Fig. 4*, the NS-AHP method has the highest correlation with expert consensus ($\rho \approx 0.9, \tau \approx 0.8$). This result shows that adding the dispersion term and using the neotrosophic structure has increased the model's compatibility with human understanding of the importance of criteria. In *Fig. 5*, the top employees (e.g., E_1 and E_2) have a steady upward trend and maintain their position over time. This indicates that the fractional model reflects the long term stability of performance better than static models (FAHP or TOPSIS) and is more suitable for evaluating continuous performance.

5 | Discussion

Integrating sparse regularization and Neutrosophic logic. By combining L_1-type dispersion constraints with Neutrosophic pairwise comparisons, this paper bridges two advanced paradigms in MCDM, allowing for the selection of salient measures while managing uncertainty. Dynamic Modeling via Fractional Neutrosophic Integral. Incorporating fractional integral dynamics offers a new avenue for modeling how employee performance evolves over time under uncertain influences-beyond static rankings. Rigorous proofs of existence, uniqueness, and stability. The contraction mapping argument ensures that the fractional update scheme provides a unique and stable path, which is valuable for theoretical robustness. Demonstration in a real, complex technical organization. An empirical case in an electric distribution company shows how the method works in practice, not just in simulated environments.

6 | Conclusion

This paper presented a Neutrosophic Sparse Analytic Hierarchy Process with Fractional Dynamics (NSAHFD) for evaluating employee performance under uncertainty, incompleteness, and time evolution. By integrating Neutrosophic logic, L_1 dispersion regularization, and fractional integral dynamics, the proposed method overcomes the fundamental shortcomings of traditional AHP, FAHP, and TOPSIS. We applied this method to a real case in TEDC and compared its outputs with FAHP and TOPSIS. The results confirmed that NSAHFD provides more consistent, robust, and interpretable rankings that are robust to missing data and disturbances. Sensitivity analysis also confirmed that the method performs more stably under moderate parameter variations. From a managerial perspective, this approach supports more confident decision-making in human resource management, especially in technical service environments characterized by uncertainty and incomplete knowledge. The dynamic scoring mechanism also provides longitudinal performance monitoring.

Author Contributions

All authors contributed significantly to this study. The first author was primarily responsible for the conceptualization of the research, development of the methodology, and drafting of the initial manuscript. The second author contributed to data collection, analysis, and interpretation of the results. The third author supervised the research process, provided critical revisions, and contributed to improving the scientific content and structure of the manuscript. All authors reviewed, edited, and approved the final version of the paper.

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Data Availability

The data used in this study were obtained through analytical procedures and/or simulations as described in the manuscript. Supporting data and additional materials related to the findings of this research are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article.

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