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A Markovian Analysis to Investigate Inter-Asset Market Linkages for Drawdown Transitions

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Abstract

This research investigates the stochastic transition dynamics of financial volatility utilizing a distinct-state first-order Markov chain model. While volatility clustering and regime persistence have been thoroughly investigated, there still exists a methodological gap in estimating transition behavior under time-homogeneous and random assumptions. To simulate regime shifts, the study utilizes Markovian transition mapping, which employs empirically derived transition matrices, stationary distributions, and absorption probabilities. Chi-square and likelihood-ratio tests have been used for diagnostic validation, ensuring precise stage classification and a sufficient level of constant transition probabilities. A comparative analysis of higher-order, non-homogeneous, and Hidden Markov Model (HMM) extensions reveals accurate regime-dependent and covariate-sensitive structures representing time-varying dynamics. At the same time, first-order chains adequately capture short-term persistence. The results show that predicted recovery time, first-passage probability, and sojourn lengths are all important for understanding how long assets will last and how quickly they will recover. The present research presents a robust probability framework that improves volatility forecasting, strategic planning, and portfolio restructuring, while developing a methodological basis for transforming traditional Markov processes into more adaptive stochastic models of financial risk.

Keywords: Stock market, Volatility, Markovian transition mapping.

1 | Introduction

Financial market drawdowns serve as crucial markers of negative risk, as they show the drop from peak to trough. Large drawdowns were described as statistical outliers associated with endogenous and external shocks by [1–3], and their scaling characteristic in price dynamics was further clarified by [4]. Another study [5] conducted a systematic assessment of drawdown risk measures, highlighting their importance for management beyond volatility metrics. Research established the theoretical foundations of drawdown control, connecting academic finance to practical portfolio management [6]. Inefficient bubbles were also linked to efficient drawdowns, which represent concealed market corrections [7]. Stochastic control methods for

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trading under Brownian motion were also developed [8], while researchers [9] examined drawdown risk in currency carrying trades. Riley and Yan [10] showed that maximum drawdown predicts mutual fund flows, and Cumming et al. [11] applied the drawdown concept to venture capital commitments. Drawdowns have been described as multidimensional risk events that extend beyond the domains of stocks, derivatives, and alternative assets.

Transitions between drawdown stages help enhance risk knowledge. The first Markovian technique for stage transition was introduced by Smith [12], who presented transition models for portfolio modification. Related work by Engle [13] modeled time-varying correlations for improved understanding of systemic transitions. Risk-shifting behaviors were studied by Beyhaghi and Hawley [14] and Rauh [15] for institutional portfolios. Recent works by [16–18] identify the causes of financial market transitions. While Amiri et al. [19] contrasted deep learning predictors of economic volatility, Kaur et al. [20] combined fuzzy and support vector models for optimal allocation under uncertainty. Shagerdi et al. [21] indirectly enhanced the transition-risk concept by tying governance to innovation risk. Together, these studies provide a solid framework for Markovian modeling of drawdown transitions in mixed-asset portfolios.

The research combines Markovian transition analysis across multiple regimes, drawing on previous studies in risk modeling and drawdown analysis. This captures the dynamic changes in the assets of cryptocurrency, gold, and equity markets, as well as static risk measurement. The research aims to gather information on fluctuations over time varying between mild, moderate, severe, and crash regimes. The study improves the overall understanding of inter-class stability using transition matrices and visualizing regime dynamics. The work also reveals comparative analysis and associated risk implications for diversified portfolio management.

1.1 | Motivations

Despite several studies on volatility modeling, risk measurements, and portfolio optimization, there exists a significant gap in our understanding of the dynamic behavior of drawdowns across asset classes. Traditional models focus primarily on volatility clustering, correlation shifts, and crash prediction, but they often overlook the market's evolution across the multiple stages of decline and recovery. Few studies directly model drawdown transitions using Markov processes, which capture the probability of moving between market states such as mild, moderate, severe, and crash. Limited comparative evaluations across asset classes of equities, cryptocurrencies, and commodities increase the complexity of measurement for systemic risk and portfolio stability. Therefore, the use of Markovian analysis for downturn analysis provides a comprehensive understanding of the potential for market stability and crash risk.

Despite advancements in volatility modeling, tail-risk characterization, and portfolio optimization, studies addressing the dynamic evolution of drawdowns through state-to-state transitions remain limited. Few studies specifically estimate drawdown transition probabilities, expected time-to-crash, or recovery durations within a Markovian framework, which limits understanding of persistence and regime-switching behavior across asset classes [1], [5], [6]. Normal transition and time-varying correlation models [12], [13] established the conceptual foundations for dynamic allocation; however, these approaches seldom address drawdown-state dynamics. The Markovian literature in finance [22–24] has mainly focused on volatility regimes, credit migration, and return dependencies, rather than on explicit drawdown transition tables, such as transition matrices, absorption probabilities, and fundamental-matrix-based expected times. The swift rise of cryptocurrencies and their significant drawdown persistence [22], [24] necessitates a cohesive Markovian analysis encompassing stocks, gold, and crypto. This approach measures comparative strength and provides actionable tables (transition matrices, absorption probabilities, projected time-to-crash/recovery) that are presently missing in the literature [5], [16].

1.2 | Research Objectives

The current work aims to fulfil the following research objectives.

I. To evaluate transition probabilities among drawdown states with a Markov chain model.

II. To analyze the decline patterns of equity indices, cryptocurrencies, and gold.

III. To derive portfolio-level implications from state transitions and persistence.

1.3 | Contributions

This research offers several contributions to theory and practice.

I. It strengthens the growing financial risk literature by integrating Markovian modeling with drawdown-based analytics, providing a probabilistic and interpretable framework for evaluating asset-class stability and the probability of crashes.

II. This research distinguishes itself from previous studies that focus on volatility clustering [13] or return correlations [22] by explicitly modeling state transitions of drawdowns across various asset classes, including NIFTY50, S&P 500, BTC-USD, ETH-USD, and Gold, using first-order Markov chains.

III. This study uses transition matrices to calculate absorption probabilities and estimate expected durations for crashes and recoveries, providing empirical evidence on the evolution of assets across mild, moderate, severe, and crash states.

IV. The framework enables a comprehensive analysis of persistence and recovery dynamics within and across markets, thereby strengthening the understanding of comparative stability between traditional and digital assets [1], [5], [6].

V. This research helps improve risk behavior analysis, regime prediction, and strategic asset allocation by integrating theoretical Markovian processes with actual drawdown behavior.

The remainder of this manuscript is organized as follows. Section 2 discusses related studies. Section 3 delineates the research methodology. The significant findings of the data analysis are reported in Section 4. Section 5 highlights the inferences drawn from the findings, and finally, Section 6 concludes the paper and highlights some future scopes.

2 | Literature Review

2.1 | Drawdown Risk and Portfolio Resilience

Prior research indicates the need to include drawdown control, adaptive optimization, and resilience evaluation in risk analytics and portfolio management. An adaptive reinforcement learning method by Almahdi and Yang [25] optimizes portfolios while minimizing projected maximum drawdown. Yang and Zhong [26] developed a risk-based dynamic allocation to combat drawdowns, whereas Nystrup et al. [27] proposed multi-period optimization with drawdown constraints. Scholars [5] conducted a comparative analysis of dynamic drawdown control mechanisms in stochastic programming, finding that drawdown-sensitive strategies outperform conventional risk measures in terms of tail-risk exposure. Similarly,Goldberg and Mahmoud [6] developed a theoretical framework that connects drawdown processes to coherent risk indicators, highlighting their interpretability and consistency in portfolio evaluation. Van Hemert et al. [28] investigated drawdown-based utility functions to evaluate investor behavior during protracted downturns, underscoring the importance of psychological and resilience factors in decision-making. Beyond portfolio optimization, Liedtke [29] examined insurance investment vulnerabilities during COVID-19, Gasser et al. [30] examined energy system resilience. Allen [31] exhibited the diversification of downside risk metrics during financial crises, while Sukma and Namahoot [32] improved algorithmic trading profitability using multi-indicator optimization. These findings suggest the growing need for drawdown-sensitive, resilience-focused financial system management during unstable, unpredictable, and dynamic regime transitions.

2.2 | Markov Processes in Finance

Markovian modeling has been necessary for understanding stochastic transitions in financial systems. [22], [33–35] provided the groundwork for regime shift and volatility persistence research. [36–38] extended Markov and decision processes for financial applications, whereas Bassler et al. [39] integrated nonlinear diffusion with Hurst exponent analysis. Recent works, such as Pourmoradi et al. [40] and Azimi et al. [41], utilize these models for stochastic control and credit risk evaluation. Finance and computational intelligence can also be linked using empirical studies. [42–44] integrate machine learning, neural networks, and IoT to improve predictive accuracy and operational efficiency. Further advances in methodology include [45] on fuzzy performance analysis, [46] on fuzzy optimization, and [47] on multi-criteria evaluation, strengthening decision-making. [48–50] use DEA, simulation, and neutrosophic logic to assess resilience in complex systems, validating Rasinojehdehi and Najafi [51] and Azimi and Chen [52].

2.3 | Markov Models in Cryptocurrencies and Multi-Asset Contexts

Recent research demonstrates advances in crypto and mixed-asset modeling through optimization, AI, and econometrics. Hybrid asset interaction models were studied Mba and Mwambi [53], Petukhina and Sprünken [54], and digital-asset strategies were evaluated Pankwaen et al. [55], while Pankwaen et al. [55] developed a multiple-asset trading optimization system. Roshanpour et al. [56] forecast Bitcoin, gold, and crude oil using a GA-optimized self-attention LSTM. Ntare et al. [57] investigated portfolio diversification for linked assets, whereas Novykov et al. [58] conducted a comprehensive assessment of deep learning in investment management. Bhatia and Bedi [59] examined the causal relationship between cryptocurrencies and emerging indices, and Song et al. [60] used reinforcement learning for portfolio control. Additional contributions include Mamplata et al [61] on dynamic metals modeling, Moodley et al. [62] on sentiment-driven co-movements, Arsi et al. [63] on crypto risks, Schellinger [64] on portfolio optimization, Li et al. [65] on LLM-based investor benchmarking, and Yamaguchi [66] on market structural shifts.

2.4 | Identified Research Gaps

Prior literature highlights significant gaps in multi-asset and cryptocurrency models, despite some improvements. Limited research on Markov chains for drawdown transitions has led to an inadequate examination of market risk temporal dynamics. Lack of a consistent comparison framework for asset classes (equity, gold, and crypto) has further complicated investment decision-making. Present research does not employ transition probability matrices, absorption probabilities, or estimates of expected time-to-crash or recovery to assess resilience and persistence during market downturns. The current research examines individual asset drawdown behavior, predicts market crash behavior, and contributes to the existing literature on probabilistic risk assessment frameworks for volatile and dynamic financial systems.

3 | Methodology

3.1 | Data Collection

Current work focuses on five significant financial assets that represent different market classes. These include the S&P 500 (a U.S. equity benchmark) and the NIFTY 50 (an Indian equity benchmark). Bitcoin (BTC-USD) and Ethereum (ETH-USD) were selected for their market dominance in the cryptocurrency market. In contrast, gold serves as a traditional haven and hedging asset. Results of the current analysis highlight a comprehensive cross-market examination of drawdown and recovery patterns.

The data used in this study was obtained from the Yahoo Finance API using the yfinance library on Google Colab, which is a Python package. Open, high, low, close, and adjusted close values, as well as historical daily price data, were automatically retrieved using this method. Data extraction, preprocessing, and analysis were carried out in an efficient cloud-based environment with no local computational limits using the Google Colab platform. The Yfinance API provides consistent, reproducible financial data for all specified assets, including NIFTY50, S&P 500, BTC-USD, ETH-USD, and Gold. This approach enables transparent data collection for Markovian modeling of drawdown transitions across several asset classes (see *Table 1*).

Table 1. Descriptive statistics (source: author analysis).

Assets	count	mean	std	min	25%	50%	75%	max
NIFTY50	4409	10653.7	6076.85	2524.2	5655.9	8629.15	14819.1	26216.1
SP500	6459	2213.5	1374.48	676.53	1207.05	1514.68	2845.4	6502.08
BTC-USD	4010	24696.5	29299.5	178.1	2482.77	10251.1	39198.3	123344
ETH-USD	2861	1607.62	1255.5	84.308	316.716	1599.48	2561.07	4831.35
Gold	6278	1215.86	660.471	255.1	645.2	1244.35	1661.3	3638.1

Daily closing prices and computed log returns: The analysis employs daily closing prices for each asset, from which log returns are computed to capture proportional changes over time.

The return series is derived using the formula:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \tag{1}$$

where P_t and P_{t-1} represent the closing prices at times t and t–1, respectively.

This transformation stabilizes variance, normalizes scale differences, and facilitates statistical modeling of return dynamics. These log returns form the foundational input for the Markov chain–based drawdown transition analysis.

3.2 | Drawdown Calculation and State Classification

The drawdown at the time t is mathematically represented as

$$DD_t = \frac{\max(P_0, P_1, \dots, P_t) - P_t}{\max(P_0, P_1, \dots, P_t)}. \tag{2}$$

This formulation measures the relative decrease of the asset from its historical maximum, presenting a standardized measure of downside risk. This metric helps with cross-asset comparisons by representing losses as a percentage of the peak value rather than in absolute terms. This study employs the drawdown series as the primary input for categorizing market states (mild, moderate, severe, crash) and for estimating transition probabilities within a Markovian framework.

Drawdown magnitudes can be distinguished into four discrete regimes to capture varying levels of market stress: Mild (0–5%), moderate (5–10%), severe (10–30%), and crash (>30%). Each stage of drawdown denotes a unique market condition that reflects the magnitude of the price decline relative to earlier peaks. These regimes describe various Markov states, which are probabilistic models of market phase transitions. This state-based classification allows for investigation of the likelihood of crashes, recovery time, and regime persistence. In addition, this creates a systematic basis for calculating transition matrices and balanced-state probabilities in later modeling.

3.3 | Markov Chain Formulation

We model drawdown dynamics as a discrete-time, finite-state Markov chain to capture probabilistic transitions between drawdown regimes. The construction here outlines the assumptions, notation, core equations, estimation methods, and key derived quantities used in the analysis. Let the finite state space be

$$\mathcal{S} = \{1, \dots, K\} = \{\text{Mild, Moderate, Severe, Crash}\},$$

where each state corresponds to a drawdown interval (e.g., Mild $=$ 0-5%, …, Crash $> 30\%$).

Under the first order (time-homogeneous) Markov assumption,

$$\Pr(X_{t+1} = j \mid X_t = i, X_{t-1}, \dots) = \Pr(X_{t+1} = j \mid X_t = i) = P_{ij}, \tag{3}$$

and the transition matrix is

$$P = [P_{ij}]_{i,j\in\mathcal{S}}, P_{ij} \geq 0, \sum_j P_{ij} = 1 \forall i. \tag{4}$$

From an observed sequence $(X_0, \dots, X_T)$ count transitions $N_{ij} = \sum_{t=0}^{T-1} 1\{X_t = i, X_{t+1} = j\}$. The empirical (MLE under multinomial rows) estimator is

$$\widehat{P}_{ij} = \frac{N_{ij}}{\sum_j N_{ij}}. \tag{5}$$

n-step probabilities follow using Chapman–Kolmogorov,

$$P^{(n)} = P^n, P_{ij}^{(n)} = \Pr(X_{t+n} = j \mid X_t = i). \tag{6}$$

If the chain is irreducible and aperiodic, a unique stationary vector π exists solving

$$\pi = \pi P, \sum_i \pi_i = 1\,, \tag{7}$$

which gives long-run proportions of time spent in each drawdown regime.

If Crash is treated as absorbing, the partition $P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$. The fundamental matrix $N = (I - Q)^{-1}$ yields expected steps in transient states and absorption-related metrics: expected time to absorption from state i is $(N1)_i$, and absorption probabilities are $B = NR$.

First-passage probabilities $f_{ij}^{(n)} = \Pr(\tau_{ij} = n \mid X_0 = i)$and mean sojourn times in the state i (empirical average run lengths) can be computed to assess persistence.

Assumptions:

Markovian framework adopted in this study relies on three fundamental assumptions: first-order memoryless dynamics, time-homogeneity (constant transition probability matrix P), and appropriately defined state discretization. To evaluate the suitability of these assumptions, the basic model is compared to higher-order or non-homogeneous alternatives using goodness-of-fit tests, such as likelihood-ratio tests and row-wise chi-square tests. Extensions of the basic model are needed when empirical diagnostics point to possible violations, such as noisy observations, state-dependent persistence, or non-constant transition probabilities. While non-homogeneous Markov chains (P_t) with covariate-dependent logits capture temporal variability, Hidden Markov Models (HMMs) are used to control latent state uncertainty and observation noise. Furthermore, enabling state-dependent sojourn distributions, semi-Markov models provide flexibility and allow a more comprehensive characterization of holding times and transition dynamics in complex financial systems.

3.4 | Transition Probability Estimation

The analysis utilises a first-order Markov chain assumption, implying that the likelihood of moving to a future drawdown state depends solely on the current state and thus exhibits the memoryless property. Transition probabilities have been computed as

$$P_{ij} = \frac{N_{ij}}{\sum_j N_{ij}},$$

where N_{ij} represents the observed frequency of transitions from state i to state j. This approach allows the construction of a transition probability matrix that captures the likelihood of movement between different drawdown regimes (e.g., mild → moderate, moderate → crash). It forms the foundation for analyzing market stability, persistence, and regime-switching dynamics within the Markovian framework.

3.5 | Stationary Distribution Analysis

The stationary distribution shows the long-term equilibrium probability of residual assets in each drawdown stage, using the Markov chain framework, and also represents each asset's balance-state vulnerability and resilience attributes.

The stationary distribution vector π satisfies the condition $\pi P=\pi$, where P is the transition probability matrix and $\sum_i \pi_i = 1$. The value of this vector reflects the proportion of time an asset will likely spend in each regime (mild, moderate, severe, or crash) as the number of transitions nears infinity. By measuring this distribution, the dominant market regimes and the comparative persistence across asset classes can be determined. A higher volume in "severe" or "crash" stages signifies increased systemic fragility and persistent drawdowns, whereas assets with elevated stationary probabilities in "mild" or "moderate" stages reflect structural stability.

3.6 | Absorption Probabilities and Expected Duration Metrics

This study employs absorption probabilities and projected duration estimates to evaluate the likelihood and duration of drawdown states within a Markovian framework. Using a partitioned transition matrix, the model analyzes the transition dynamics among mild, moderate, and severe drawdown states, treating the "crash" regime as an absorbing state. The fundamental matrix $N = (I - Q)^{-1}$ indicates the frequency of trips to temporary states and the time-to-crash. At the same time, the absorption probability matrix $B = NR$ determines the chances of moving to the Crash state from any given regime. The metrics demonstrate the resilience and fragility of the NIFTY 50, S&P 500, BTC-USD, ETH-USD, and Gold—by examining their deterioration and rebounds. Empirical computation and cross-asset comparison of these metrics provide a probabilistic foundation for managing portfolio drawdown risk and optimizing asset allocation under dynamic market conditions.

3.7 | Comparative Risk Interpretation

Comparative Risk Interpretation evaluates credit states' dynamic behavior using basic Markov chain metrics such as transition probabilities, steady-state distributions, absorption probability, and anticipated duration metrics. Transition probabilities reflect the likelihood of moving between credit states, while steady-state distributions identify long-term equilibrium conditions across risk categories. Absorption probabilities suggest systemic vulnerability by highlighting the potential of moving into an irrevocable or default condition. Anticipated duration is the average time a credit organization remains in a specific risk category before a change. The research highlights that high-risk states exhibit higher expected lifespans and increased absorption tendencies, whereas low-risk states maintain stability and return to equilibrium more rapidly. The quantitative analysis demonstrates the importance of early detection methods in credit risk management, highlighting structural deficiencies in financial resilience.

4 | Results and Discussion

4.1 | Asset Transition Matrix

For BTC-USD drawdown regimes, the image illustrates the condition during the transition probability. As observed, the high probability values along the diagonal suggest that the asset is likely to remain in its current drawdown state from one period to the next, a phenomenon known as state persistence. 'crash' depicts a scenario with a high probability of occurrence, where this situation becomes more noticeable. The off-diagonal elements clearly map possible pathways along which a drawdown can deepen or recover over time, demonstrating the lower probabilities of shifting between various severity states (see *Fig. 1*). ETH-USD drawdown states, state transitions, and durability have been highlighted in the given matrix (see *Fig. 2*). While all the possible situations of crash, severe, moderate, and mild with increasing probabilities have been highlighted in the diagonal of the matrix, the 'crash' suggests prolonged and severe downturns. Unlikely: weaker transitions are visible in the off-diagonal parts, indicating a transition to either a severe crisis or a gradual return to a milder state over time (see *Fig. 1*). This matrix illustrates Gold's drawdown state transitions, revealing its characteristic stability (see *Fig. 3*).

The diagonal offers the highest likelihood of sustaining its current state, especially during "Mild" and "Moderate" drawdown regimes. This reveals the highly resilient and volatile feature of Gold, as extreme market conditions do not much influence it. The minimal off-diagonal transition probabilities reinforce Gold's position as a haven asset, as observed during the "crash" state, under protection during turmoil, with a low risk of extreme, persistent losses (see *Fig. 1*). This matrix depicts the drawdown state transitions for the NIFTY50 index. The central value on the diagonal suggests the current drawdown state will persist, with strong state persistence, particularly during the "crash state," which has a high likelihood of continuing. In contrast, the off-diagonal elements exhibit transitions between different severity levels with the possibility of either improvement or decline. Collectively, the indices suggest significant downturns, with some stability and an elevated chance of survival or reverting to milder states (see *Fig. 1*).

Crash (>30%), severe (10-30%), moderate (5-10%), and mild (0-5%), the SP500 Drawdown Matrix predicts a state transition. Dark blue cells indicate diagonal dominance, suggesting the market can trade in each state during the next term. Moderate to mild or severe to crash shifts between nearby states are possible, but the softer colors indicate the rarity of events. The matrix demonstrates market stability within each regime and that catastrophic crashes are rare transitions, emphasizing the need to focus on early decline (see *Fig. 1*, *Table 2*).

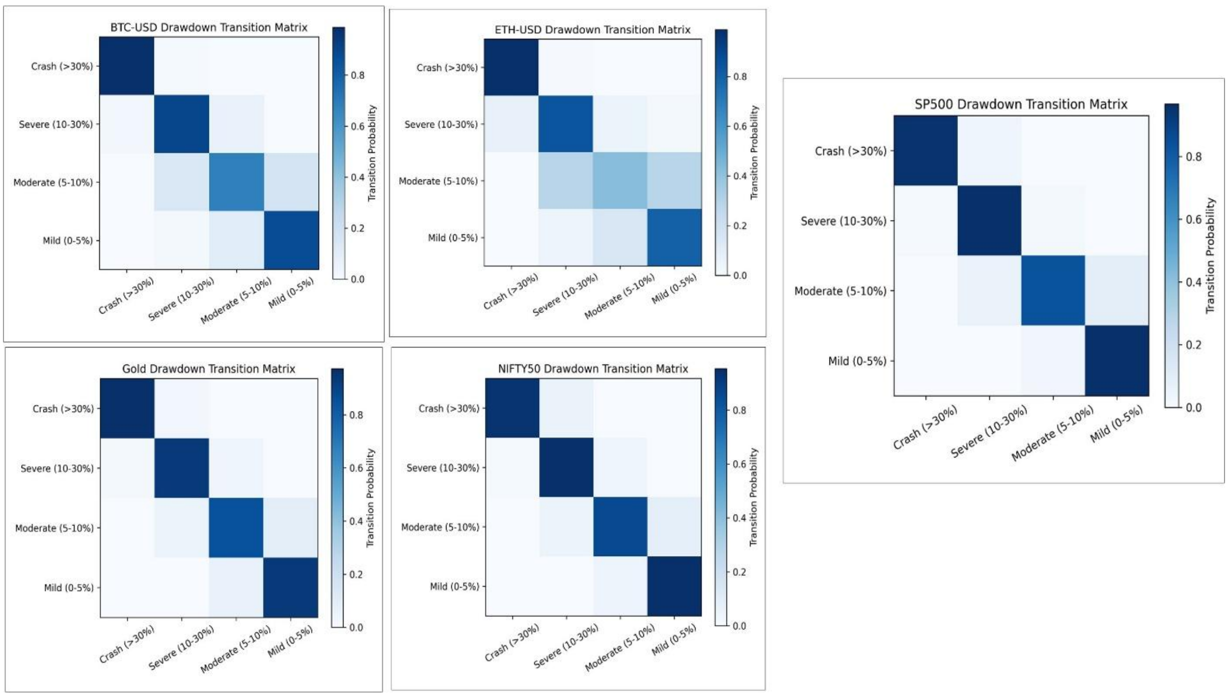

Fig. 1. Asset transition matrix.

Table 2. Transition matrices.

		Crash (>30%)	Severe (10-30%)	Moderate (5-10%)	Mild (0-5%)
NIFTY50	Crash (>30%)	0.938	0.062	0	0
	Severe (10-30%)	0.012	0.949	0.04	0
	Moderate (5-10%)	0	0.052	0.86	0.088
	Mild (0-5%)	0	0	0.046	0.954
SP500	Crash (>30%)	0.955	0.045	0	0
	Severe (10-30%)	0.014	0.964	0.022	0
	Moderate (5-10%)	0	0.063	0.839	0.098
	Mild (0-5%)	0	0	0.032	0.968
BTC-USD	Crash (>30%)	0.988	0.012	0	0
	Severe (10-30%)	0.029	0.902	0.066	0.003
	Moderate (5-10%)	0	0.146	0.68	0.174
	Mild (0-5%)	0	0.017	0.107	0.877
ETH-USD	Crash (>30%)	0.989	0.011	0	0
	Severe (10-30%)	0.073	0.848	0.061	0.017
	Moderate (5-10%)	0	0.288	0.425	0.288
	Mild (0-5%)	0	0.049	0.148	0.803
Gold	Crash (>30%)	0.974	0.026	0	0
	Severe (10-30%)	0.022	0.934	0.044	0.001
	Moderate (5-10%)	0	0.052	0.852	0.096
	Mild (0-5%)	0	0	0.065	0.935

4.2 | Stationarity Distribution of All Assets

The stationary distribution of drawdown states shows the long-term likelihood of each asset being in a given state, as shown in this table. The "crash" state is mainly dominated by both cryptocurrencies (BTC and ETH), with 81% probability for ETH and 55% for BTC, respectively, indicating their inherent risk. On the other hand, traditional assets (NIFTY, SP500, Gold) are widely held in "mild" or "severe" states, with gold and equities having a significantly lower risk of a crash. Findings suggest that cryptocurrencies are considerably more vulnerable to substantial, prolonged declines than traditional markets (see *Table 3*).

Table 3. Stationary distribution of all assets.

Region	NIFTY50	SP500	BTC-USD	ETH-USD	Gold
Crash (>30%)	0.055173	0.11814803	0.5521	0.807775984	0.207742552
Severe (10-30%)	0.293876	0.36559306	0.2233	0.120359323	0.25155329
Moderate (5-10%)	0.223656	0.1269743	0.0906	0.024883952	0.21746057
Mild (0-5%)	0.427296	0.38928461	0.1339	0.046980741	0.323243588

4.3 | Absorption Probabilities

The chances of a "crash" condition (a decline of 30% or more) (refer to *Table 4*), reflect the long-term risk of each asset experiencing a significant downturn. Cryptocurrencies represent a significantly higher risk, with ETH-USD having an 81% chance and BTC-USD holding a 55% chance of a crash experience. In contrast, traditional assets (the NIFTY50 and the SP500) have potentially fewer crash probabilities (6% and 12%, respectively), as compared to gold, which retains a moderate risk profile (21%). This clearly illustrates cryptocurrency's extreme inherent volatility and risk as compared to more stable traditional investments (see *Table 4*).

Table 4. Absorption probabilities.

	Crash Probability (>30%)
NIFTY50	0.0552
SP500	0.1181
BTC-USD	0.5521
ETH-USD	0.8078
Gold	0.2077

4.4 | Absorption Steps

The table (see *Table 5*) illustrates the probability (1.0 for most states) that an asset will transition to a "mild" or "crash" state from any other starting state. For all assets, the probability of accessing the "mild" condition from any other state is around 100%, while the likelihood of entering the "crash" state is 0. This suggests that, despite severe drawdowns, all these markets have a historical tendency to eventually recover to mild or minimal drawdowns over the long term, with no permanent absorption into a collapse state (refer to *Table 5*).

Table 5. Absorption steps.

	NIFTY50	SP500	BTC-USD	ETH-USD	Gold
Crash (>30%)	0	0	0	0	0
Mild (0-5%)	0.9999	1	1	1	1
Moderate (5-10%)	1	0.9999	1	1	0.9999
Severe (10-30%)	1	0.9999999	1	1	0.9999

4.5 | Expected Recovery Time (First Passage Probabilities)

The table (refer to *Table 6*) shows the expected time frame for the recovery of assets from a drawdown state to the "mild" (0–5%) recovery state. Assets, including NIFTY50, SP500, BTC-USD, ETH-USD, and gold, are expected to recover from any level of downturn (crash, severe, or moderate) to a median state within a single month. The findings add to the historical strength of these markets, highlighting that observed declines tend to be followed by rapid recovery to previous levels with little drawdown. This suggests a persistent trend of rapid recovery across both traditional and cryptocurrency assets (see *Table 6*).

Table 6. Expected recovery time.

	NIFTY50	SP500	BTC-USD	ETH-USD	Gold
Crash (>30%)	1	1	1	1	1
Mild (0-5%)	0	0	0	0	0
Moderate (5-10%)	1	1	1	1	1
Severe (10-30%)	1	1	1	1	1

The table shows the likelihood that an asset will first enter a drawdown state at a specified future phase (1-10) (see *Table 7*). A significant insight is the high probability (0.938-0.989) of a "crash" occurring in the very first stage if the asset is not already in one, underscoring the sudden onset of severe downturns. Other states' probabilities often remain modest and tend to diminish over time. Furthermore, cryptocurrencies, in particular ETH-USD, exhibit a higher likelihood of severe and moderate states in earlier stages, suggesting a tendency toward sudden, substantial price drops compared to more stable assets such as gold (see *Table 7*).

Table 7. First passage probabilities.

Assets	Regions	1	2	3	4	5	6	7	8	9	10
NIFTY50	**Crash (>30%)**	**0.938**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	Severe (10-30%)	0.012	0.011	0.01	0.01	0.01	0.009	0.009	0.008	0.008	0.008
	Moderate (5-10%)	0	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.003
	Mild (0-5%)	0	0	0	0	0	0	0	0	0	0.001
SP500	Crash (>30%)	0.955	0	0	0	0	0	0	0	0	0
	Severe (10-30%)	0.014	0.014	0.013	0.013	0.013	0.012	0.012	0.011	0.011	0.011
	Moderate (5-10%)	0	0.001	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.004
	Mild (0-5%)	0	0	0	0	0	0	0	0	0.001	0.001
BTC-USD	Crash (>30%)	0.988	0	0	0	0	0	0	0	0	0
	Severe (10-30%)	0.029	0.026	0.024	0.022	0.02	0.019	0.018	0.017	0.016	0.015
	Moderate (5-10%)	0	0.004	0.007	0.008	0.009	0.01	0.01	0.01	0.011	0.011
	Mild (0-5%)	0	0	0.001	0.002	0.003	0.004	0.005	0.006	0.006	0.007
ETH-USD	Crash (>30%)	0.989	0	0	0	0	0	0	0	0	0
	Severe (10-30%)	0.073	0.062	0.054	0.048	0.042	0.038	0.035	0.032	0.029	0.027
	Moderate (5-10%)	0	0.021	0.028	0.03	0.03	0.03	0.03	0.029	0.029	0.028
	Mild (0-5%)	0	0.004	0.009	0.014	0.018	0.021	0.023	0.025	0.026	0.026
Gold	Crash (>30%)	0.974	0	0	0	0	0	0	0	0	0
	Severe (10-30%)	0.022	0.02	0.019	0.018	0.017	0.016	0.015	0.014	0.013	0.013
	Moderate (5-10%)	0	0.001	0.002	0.003	0.003	0.004	0.004	0.004	0.004	0.005
	Mild (0-5%)	0	0	0	0	0	0.001	0.001	0.001	0.001	0.001

4.6 | Sojourn Times

The expected sojourn time is the length of time that an asset stays in a particular drawdown stage after entering it, as depicted in the table (see *Table 8*). Results of the experiment reflect the robustness of Cryptocurrencies in the "crash" state, with ETH-USD and BTC-USD remaining for 92 and 85 periods on average, respectively, much longer than traditional assets that include NIFTY50 (16) or gold (38). In contrast, cryptocurrencies remain in the "mild" category for a shorter duration (5-8 periods) than equities and gold (21-31 periods) (see *Table 8*).

Table 8. Sojourn Times.

Regions	NIFTY50	SP500	BTC-USD	ETH-USD	Gold
Crash (>30%)	16.1333333	22.4411765	85	92.08	38.3529412
Severe (10-30%)	19.530303	27.4534884	10.1590909	6.59615385	15.0380952
Moderate (5-10%)	7.16058394	6.21212121	3.12068966	1.73809524	6.75742574
Mild (0-5%)	21.7931034	31.425	8.10447761	5.07142857	15.3712121

4.7 | Final Comparison Table

A comprehensive investigation suggests a clear division across asset classes. Cryptocurrencies (BTC and ETH) exhibit extreme volatility, as evidenced by their high standard deviations, maximum values, and a significantly increased probability of experiencing a crash (55% and 81%). The expected duration of cryptocurrency downturns after a collision can be extremely substantial (85 to 92 periods), implying significant, extended declines. In comparison, traditional assets such as NIFTY, SP500, and Gold exhibit markedly lower crash probabilities and are expected to remain in a stable "Mild" state for an extended period. Despite all assets indicating a fast estimated drop under any circumstance (1 step), the metric is greatly influenced by the significant volatility of cryptocurrencies, outweighing the overall stability and security of traditional markets (see *Table 9*).

Table 9. Comparison table.

Assets	mean	std	min	max	Crash Probability (>30%)	Expected Steps to Crash from Severe (10-30%)	Expected Steps to Crash from Moderate (5-10%)	Expected Steps to Crash from Mild (0-5%)	Expected Sojourn Time in Crash (>30%)	Expected Sojourn Time in Severe (10-30%)	Expected Sojourn Time in Moderate (5-10%)	Expected Sojourn Time in Mild (0-5%)
NIFTY50	10653.72	6076.85	2524.2	26216.1	0.055	1	1	1	16.133	19.53	7.161	21.793
SP500	2213.499	1374.48	676.53	6502.08	0.118	1	1	1	22.441	27.453	6.212	31.425
BTC-USD	24696.47	29299.5	178.103	123344	0.552	1	1	1	85	10.159	3.121	8.104
ETH-USD	1607.616	1255.5	84.308	4831.35	0.808	1	1	1	92.08	6.596	1.738	5.071
Gold	1215.862	660.471	255.1	3638.1	0.208	1	1	1	38.353	15.038	6.757	15.371

The flowchart highlights an extensive Markovian analysis of financial market downturns. It uses Yfinance to aggregate asset price data and preprocess daily price data to predict returns and drawdowns to classify market regimes by severity. Evaluating transition probabilities to create the Markov chain yields the transition matrix, the stationary distribution, and the absorption probabilities. The ground interpretation focuses on the first passage and sojourn times. It provides information on recovery dynamics and the expected time to a crash. Finally, probabilistic measures can assess portfolio risk, stress test, and allocate strategic assets (see *Fig. 2*).

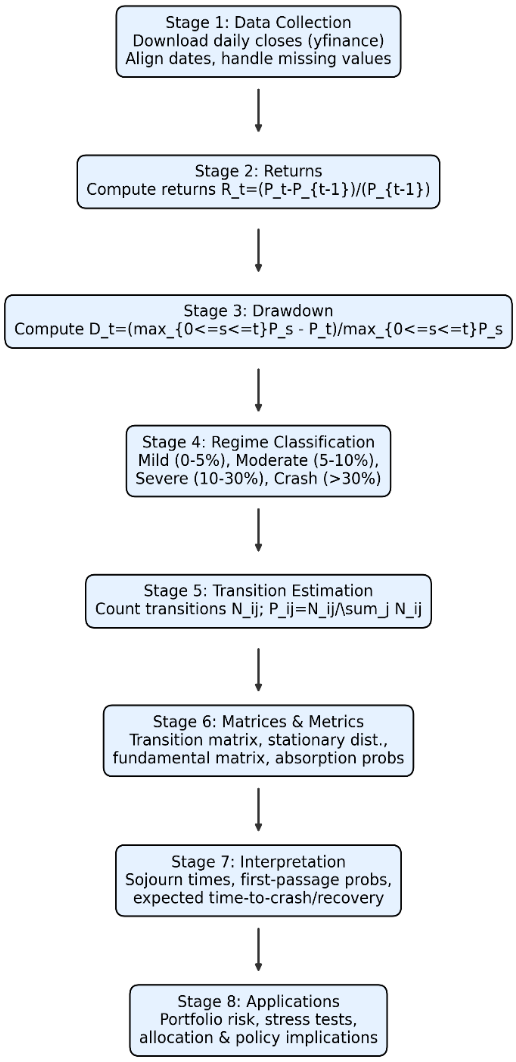

Fig. 2. Comprehensive framework for Markovian analysis.

5 | Business and Portfolio Implications

This paper offers several insights for managerial decision-making and policy formulations

I. Transition-driven crash risk indicators from Markov analysis provide a more adaptable perspective than volatility-based metrics.

II. It enables the construction of dynamic portfolios by probabilistically allocating weights across asset classes that correspond with their anticipated resilience and stability.

III. It improves investment timing by indicating regime shifts, enabling proactive portfolio rebalancing across mild to severe market situations.

IV. The approach provides data-driven insights into long-term equilibrium behaviors, recovery probabilities, and regime persistence that strengthen policy and investment initiatives.

6 | Conclusion and Suggestions

The study illustrates that Markovian transition mapping accurately represents the probabilistic dynamics of financial market drawdowns across various asset classes. It demonstrates apparent regime persistence and transition probabilities, indicating that traditional volatility metrics are unable to assess crash likelihood and recovery duration adequately. The integration of stationary distributions, absorption probabilities, and expected time to crash or recovery yields interpretable risk indicators for analyzing portfolio resilience. The framework's methodological strength is evident in its systematic mapping of drawdown regimes into Markov states, which facilitates cross-asset comparison and improves the predictive and diagnostic capabilities of risk management models. Additionally, drawdown-based Markov modeling provides new interpretive dimensions for mixed-asset portfolios.

Future studies in this direction can strengthen this work by testing more regime-dependent volatility across varying degrees of market turbulence. Integrating non-homogeneous Markov models could also lead to the evolution of standard benchmarks for transition estimation and to the development of accurate early warning systems. Further work can also link cross-market transmission to systemic risk to enhance understanding of inter-asset linkages during financial instability and to build a better perspective on market resilience and recovery.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

All data generated or analyzed during this study are included in this published article. Additional supporting data are available from the corresponding author upon reasonable request.

Authors' Contributions

R. R.: Conceptualization, Methodology, Investigation, Writing–Original Draft Preparation.

R. M.: Supervision, Validation, Formal Analysis, Writing–Review & Editing.

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This study did not involve human participants, human data, human tissue, or animals. Therefore, ethics committee approval and informed consent to participate were not required.

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